Axiomatic and Computational Aspects of Scoring Allocation Rules for Indivisible Goods

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Abstract

We define a family of rules for dividing m indivisible goods among agents, parameterized by a scoring vector and a social welfare aggregation function. We assume that agents' preferences over sets of goods are additive, but that the input is ordinal: each agent simply ranks single goods. Similarly to (positional) scoring rules in voting, a scoring vector $s = (s_1, \ldots, s_m)$ consists of m nonincreasing nonnegative weights, where s_i is the score of a good assigned to an agent who ranks it in position i. The global score of an allocation for an agent is the sum of the scores of the goods assigned to her. The social welfare of an allocation is the aggregation of the scores of all agents, for some aggregation function $\star$ such as, typically, $+$ or min. The rule associated with s and $\star$ maps a profile to (one of) the allocation(s) maximizing social welfare. After defining this family of rules, and focusing on some key examples, we investigate some of the social-choice-theoretic properties of this family of rules, such as various kinds of monotonicity, separability, envy-freeness, and Pareto efficiency. Then we focus on the computation and approximation of winning allocations.

1 Introduction

Fair division of a divisible good has put forth an important literature about *specific procedures*, either centralized [17] or decentralized [9]. Fair division of *a set of indivisible goods* has, perhaps surprisingly, been mainly addressed by looking for allocations that satisfy a series of properties (such as equity or envy-freeness) and less often by defining specific allocation rules. A notable exception is a series of papers that assume that each agent values each good by a positive number, the utility of an agent is the sum of the values of the goods assigned to her, and the resulting allocation maximizes social welfare; in particular, the *Santa Claus problem* [2] considers egalitarian social welfare, which maximizes the utility of the least happy agent. A problem with these rules is that they strongly rely on the assumption that the input is numerical. Now, as widely discussed in social choice, numerical inputs have the strong disadvantage that they suppose that interpersonal preferences are comparable. Moreover, from a practical designer point of view, eliciting numerical preferences is not easy: in contexts where money does not play any role, agents often feel more at ease expressing rankings than numerical utilities.

These are the main reasons why social choice – at least its subfield focusing on voting – usually assumes that preferences are expressed ordinally. Surprisingly, while voting rules defined from ordinal preferences have been addressed in hundreds of research articles, we can find only a few such papers in fair division (with the notable exception of matching, discussed below). Brams, Edelman, and Fishburn [6] assume that agents rank single goods and have additively separable preferences; they define a Borda-optimal allocation to be one that maximizes egalitarian social welfare, where the utility of an agent is the sum of the Borda scores of the objects assigned to her, and where the Borda score of object g_i for agent j ranges from 1 (when g_i is j's least preferred object) to m (when g_i is j's most preferred object). Unlike Brams et al. [6], Herreiner and Puppe [15] assume that agents should express rankings over *subsets* of goods, which, in the worst case, requires agents to express an exponentially large input, which should be avoided for obvious reasons.

One setting where it is common to use ordinal inputs is *two-sided matching*. But there, only one item is assigned to each agent, making this a rather different problem. This remark allows us to see fair division rules defined from ordinal inputs as a one-to-many extension of matching mechanisms. Examples of practical situations when one has to assign not a single, but several (sometimes many) items to each agent are common, and expressing quantitative utilities is not always feasible in such cases: composition of sport teams, divorce settlement, exploitation of Earth observation satellites (see [9] for more examples).

We start by generalizing Borda-optimal allocations [6] to arbitrary scoring vectors and aggregation functions. Beyond Borda, the scoring vectors we consider are k-approval (the first k objects get score 1 and all others get 0), lexicographicity (an item ranked in position k counts more than the sum of all objects ranked in positions $k+1$ to m), and quasi-indifference (for short, QI: all objects have roughly the same score, up to small differences). As for aggregation functions, we focus on utilitarianism ($\star = +$) and egalitarianism ($\star = \min$, as well as $\star =$ leximin, which in a *strict sense* is not an aggregation function). In Section 2, we define these allocation rules (we consider both resolute rules and irresolute rules), and focus on a few particular cases. Section 3 is devoted to axiomatic properties. While the properties of voting rules have been studied extensively, this is much less the case for fair allocation of indivisible goods. Perhaps the most closely related research is [12] who study the axiomatic property of *multiwinner voting rules*, with a focus on positional scoring rules, while the relationship between multiwinner rules and resource allocation is addressed in [19].

In Section 3.1, we consider *separability*, which, roughly, says that if we partition the set of agents into two subsets, A_1 and A_2, where A_i collectively gets the set G_i of goods under an optimal allocation π, and if we then consider the allocation problem restricted to A_i and G_i, then the agents in A_i will get the same set G_i of goods as in π. Section 3.2 considers *monotonicity*: if agent i gets good g under the optimal allocation π, and if the rank of g is raised in i's ranking with everything else being unchanged, will i still get g? In Section 3.3, we look at two other forms of monotonicity, named *object monotonicity* (if some good is added, will the new allocation make all agents at least as happy as before?) and *duplication monotonicity* (which is also related to "cloning" agents). Finally, in Section 3.4, we consider various *consistency* and *compatibility properties*. In Section 4, we focus on the complexity of winner determination for a few key combinations of a scoring vector and an aggregation function, considering both decision and functional problems. In Section 5, we give several approximation results, some of which make use of *picking sequences*. Section 6 discusses some open questions for future research.

2 Scoring Allocation Rules

Let $N = \{1,\ldots,n\}$ be a set of agents and $G = \{g_1,\ldots,g_m\}$ a set of indivisible goods (we will use the terms *good*, *item*, and *object* as synonyms). An *allocation* is a partition $\pi = (\pi_1,\ldots,\pi_n)$, where $\pi_i \subseteq G$ is the bundle of goods assigned to agent i. We say that allocation π *gives* g_i *to* j if $g_i \in \pi_j$.

In the general case, to compute an optimal allocation (for some notion of optimality) we would need, for every agent, her ranking over all subsets of G. As listing all (or a significant part of) the subsets of G would be infeasible in practice, we now make a crucial assumption: *agents rank only single objects*. This assumption is not without loss of generality, and has important consequences; in particular, it will not be possible for agents to express preferential dependencies between objects. Under this assumption, a *singleton-based profile* $P = (>_1,\ldots,>_n)$ is a collection of n rankings (i.e. linear orders) over G, and a *(singleton-based) allocation rule* (respectively, an *allocation correspondence*) maps any profile to an allocation (respectively, a nonempty subset of allocations). For any ranking $>$ (respectively, profile P) over G, and any subset $G' \subset G$, we will write $>_{|G'}$ (respectively, $P_{|G'}$) to denote the *restriction* of $>$ (respectively, P) to G'. Similarly, we denote the restriction of P to any subset $N' \subset N$ by $P_{|N'}$.

We now define a family of allocation rules that more or less corresponds to the family of scoring rules in voting (see, e.g., [7]).

Definition 1 *A* scoring vector *is a vector* $s = (s_1, \ldots, s_m)$ *of real numbers such that* $s_1 \geq \cdots \geq s_m \geq 0$ *and* $s_1 > 0$*. Given a preference ranking* $>$ *over* G *and* $g \in G$*, let* $rank(g, >) \in \{1, \ldots, m\}$ *denote the* rank of g under $>$. *The* utility function over 2^G induced by the ranking $>$ on G and the scoring vector s *is for each bundle* $X \subseteq G$ *defined by* $u_{>,s}(X) = \sum_{g \in X} s_{rank(g,>)}$.

A *strictly decreasing* scoring vector s satisfies $s_i > s_{i+1}$ for each $i < m$. A scoring vector is only defined for a fixed number of objects. To deal with a variable number of objects, we introduce the notion of *extended scoring vector*, as a function mapping each integer m to a scoring vector $s(m)$ of m elements. We consider the following specific extended scoring vectors:

- Borda scoring: borda $= m \mapsto (m, m-1, \ldots, 1)$,[1]
- lexicographic scoring: lex $= m \mapsto (2^{m-1}, 2^{m-2}, \ldots, 1)$,
- quasi-indifference for some extended scoring vector s: s-qi $= m \mapsto (1 + {}^{s_1(m)}/_{M}, \ldots, 1 + {}^{s_m(m)}/_{M})$, with $M \gg m \cdot \max\{s_1(m), \ldots, s_m(m)\} = m \cdot s_1(m)$, where M is an arbitrary and large integer.
- k-approval: k-app $= m \mapsto (1, \ldots, 1, 0, \ldots, 0)$, where the first k entries are ones and all remaining entries are zero.

In the following, we will often abuse notation and use scoring vectors and extended scoring vectors interchangeably, and omit the parameter m when the context is clear.

Note that quasi-indifference makes sense for settings where all agents should get the same number of objects (plus/minus one). An example of quasi-indifference scoring vector would be the one proposed by Bouveret and Lang [5], namely borda-qi $= (1 + {}^{m}/_{M}, 1 + {}^{(m-1)}/_{M}, \ldots, 1 + {}^{1}/_{M})$.

For example, let $G = \{a, b, c\}$ be a set of three goods and let two agents have the following preference profile: $(a >_1 b >_1 c,\ b >_2 c >_2 a)$. Let $\pi = (\{a\}, \{b, c\})$. Then, for the Borda scoring vector, agent 1's bundle $\{a\}$ has value 3 and agent 2's bundle $\{b, c\}$ has value $3 + 2 = 5$.

It is important to note that *we do not claim that these numbers actually coincide, or are even close to, the agents' actual utilities* (although, in some specific domains, scoring vectors could be learned from experimental data). But this is the price to pay for defining rules from an *ordinal* input (see the Introduction for the benefits of ordinal inputs). This tradeoff is very common in voting theory: the well-studied family of *scoring rules* in voting theory (including the Borda rule) proceeds exactly the same way; voters rank alternatives, and the ranks are then mapped to scores; the winning alternatives are those that maximize the sum of scores. If we aim at maximizing actual social welfare, then we have to elicit the voters' (numerical) utilities rather than just asking them to rank objects. Caragiannis and Procaccia [10] analyze this ordinal-cardinal tradeoff in voting and show that the induced distortion is generally quite low. A reviewer pointed out that this approach also can be seen as optimizing the external perception of fairness or welfare.

The individual utilities are then aggregated using a monotonic, symmetric *aggregation function* that is to be maximized. The three we will use here are among the most obvious ones: utilitarianism (*sum*) and two versions of egalitarianism (*min* and *leximin*). Leximin refers to the (strict) lexicographic preorder over utility vectors whose components have been preordered nondecreasingly. Formally, for $x = (x_1, \ldots, x_n)$, let $x' = (x'_1, \ldots, x'_n)$ denote some vector that results from x by rearranging the components of x nondecreasingly, and define $x <_{\text{leximin}} y$ if and only if there is some i, $0 \leq i < n$, such that $x'_j = y'_j$ for all j, $1 \leq j \leq i$, and $x'_{i+1} < y'_{i+1}$, and $x \leq_{\text{leximin}} y$ means $x <_{\text{leximin}} y$ or $x = y$. Let leximin denote the maximum on a set of utility vectors according to $\leq_{\text{leximin}}$. For each scoring vector s, define three *allocation correspondences*:

- $F_{s,+}(P) = \text{argmax}_\pi \sum_{1 \leq i \leq n} u_{>_i,s}(\pi_i)$,

[1] Note that the usual definition of the Borda scoring vector in voting is $(m-1, m-2, \ldots, 1, 0)$. Here, together with [6] we fix the score of the bottom-rank object to 1, meaning that getting it is better than nothing. For scoring voting rules, a translation of the scoring vector has obviously no impact on the winner(s); for allocation rules, however, it does. See Example 2.

- $F_{s,\min}(P) = \text{argmax}_\pi \min_{1\le i\le n}\{u_{>_i,s}(\pi_i)\}$, and
- $F_{s,\text{leximin}}(P) = \text{argleximin}_\pi(u_{>_1,s}(\pi_1),\ldots,u_{>_n,s}(\pi_n))$,

where $P = (>_1,\ldots,>_n)$ is a profile and $\pi = (\pi_1,\ldots,\pi_n)$ an allocation. Whenever we write $F_{s,\star}$, we mean any one of $F_{s,+}$, $F_{s,\min}$, and $F_{s,\text{leximin}}$.

Example 2 *For $n = 3$ agents and $m = 4$ goods, $G = \{a,b,c,d\}$, let $P = (c >_1 b >_1 a >_1 d,\ c >_2 a >_2 b >_2 d,\ b >_3 d >_3 c >_3 a) = (cbad,\ cabd,\ bdca)$. Then, $F_{(4,3,2,1),\text{leximin}}(P) = \{(c,ad,b)\}$ and $F_{(3,2,1,0),\text{leximin}}(P) = \{(c,a,bd)\}$. (From now on, we sometimes omit stating "$>_i$" explicitly in the preferences, and parentheses and commas in allocations.)*

Tie-breaking: Similarly as in voting theory, an *allocation rule* is defined as the composition of an allocation correspondence and a tie-breaking mechanism, which breaks ties between allocations. One particular type of deterministic tie-breaking mechanism consists in defining it from a linear order $>_T$ over all allocations,[2] or, when N and G are not both fixed, a collection of linear orders $>_T^{N,G}$ (which we still denote by $>_T$) for all possible sets of agents and goods, N and G. We write $\pi \ge_T \pi'$ for ($\pi >_T \pi'$ or $\pi = \pi'$). As in voting theory, if the output of a correspondence $F(P)$ is not a singleton, then the most prioritary allocation in $F(P)$ is selected: $F^T(P) = (T \circ F)(P) = max(>_T, F(P))$.

We do not make any assumption as to how this tie-breaking relation is defined; our results hold independently of that.

One may also wonder whether it is possible to define an anonymous tie-breaking mechanism, as is common in voting. Formally, a tie-breaking mechanism $>_T$ is *anonymous* if and only if for any permutation σ over N and any pair of allocations (π,π'), we have $\pi >_T \pi' \Leftrightarrow \sigma(\pi) >_T \sigma(\pi')$, where $\sigma(\pi)$ denotes the version of π where all shares have been permuted according to σ. In fact, the answer is negative (we omit the easy proof): There is no deterministic anonymous tie-breaking mechanism.

3 Axiomatic Properties

The properties we study in the paper are primarily defined for deterministic rules. Some of them will be immediately generalizable for correspondences, and in that case we'll also discuss whether or not they hold for correspondences. However, others do not generalize in a straightforward way to correspondences.[3] For these properties, we will leave the study of whether they hold for scoring resource allocation correspondences for further research.

3.1 Separability

Slightly reformulating Thomson [20], an allocation rule is *consistent* (we prefer to choose the terminology *"separable"*) if for any allocation problem and any allocation π selected by the rule, the allocation rule chooses the same allocation regardless of whether π is restricted to a subgroup of agents or when reapplying the rule to a "reduced problem" obtained by imagining the departure of any subgroup of the agents with their share. As the definition generalizes easily to allocation correspondences, we define it for both.

[2] This choice comes with a loss of generality, as there are tie-breaking mechanisms that are not defined this way (we thank a reviewer for this remark). Also, we rule out the possibility of randomly breaking ties.

[3] This is the case for all properties expressing that an agent prefers a set of allocations to another set of allocations (and applies, e.g., to object monotonicity); for these properties there is not a unique way of generalizing the property, unlike in voting where this is well-known, e.g., for strategy-proofness.

Definition 3 *Let $P = (>_1, \dots, >_n)$ be a profile over a set G of goods and consider any partition of the set N of agents into two sets, N^1 and N^2, i.e., $N^1 \cup N^2 = \{1, \dots, n\}$ and $N^1 \cap N^2 = \emptyset$. Let $\pi = (\pi_1, \dots, \pi_n)$ and for $j \in \{1,2\}$, let $G^j = \bigcup_{i \in N^j} \pi_i$. An allocation rule F satisfies* separability *if for each P and π, $F(P_{|N^1,G^1}) = \pi^1$ and $F(P_{|N^2,G^2}) = \pi^2$, where π^i denotes the restriction of π to N^i and G^i. An allocation correspondence F satisfies* separability *if for each P and π, $\pi \in F(P)$ if and only if $\pi^1 \in F(P_{|N^1,G^1})$ and $\pi^2 \in F(P_{|N^2,G^2})$. Also, we say that a tie-breaking priority T is* separable *if $\pi^1 \geq_T \pi'^1$ and $\pi^2 \geq_T \pi'^2$ implies $\pi \geq_T \pi'$.*

Unfortunately, it looks like almost all our rules violate separability. We give a counterexample that works for many choices of $(s, \star)$.

Example 4 *Let $m = 9$, $n = 3$, $\star \in \{+, \min, \text{leximin}\}$, and s be a strictly decreasing vector. Consider the preference profile $P = (g_1g_4g_3g_6g_8g_7g_2g_5g_9,\ g_2g_5g_1g_8g_7g_3g_4g_6g_9,\ g_3g_6g_1g_2g_9g_4g_5g_7g_8)$. $F_{s,\star}(P)$ consists of the unique allocation $\pi = (g_1g_4g_8,\ g_2g_5g_7,\ g_3g_6g_9)$ for $\star \in \{\min, \text{leximin}\}$, and $F_{s,+}(P)$ consists of the unique allocation $\pi' = (g_1g_4, g_2g_5g_7g_8, g_3g_6g_9)$. The restriction of P to agents $\{1,2\}$ and goods $\{g_1, g_2, g_4, g_5, g_7, g_8\}$ is $P' = (g_1g_4g_8g_7g_2g_5,\ g_2g_5g_1g_8g_7g_4)$. For $\star \in \{\min, \text{leximin}\}$, $F_{s,\star}(P')$ consists of the unique allocation $(g_1g_4g_7,\ g_2g_5g_8) \neq (g_1g_4g_8,\ g_2g_5g_7)$, and $F_{s,+}(P')$ consists of the unique allocation $(g_1g_4g_7g_8, g_2g_5) \neq (g_1g_4, g_2g_5g_7g_8)$.*

We conjecture that (perhaps under mild conditions on s and $\star$), no positional scoring allocation rule is separable.

3.2 Monotonocity

The monotonicity properties below state that if an agent ranks a received good higher, all else being equal, then this agent does not lose this good (monotonicity) or still receives the same bundle (global monotonicity).

Definition 5 *An allocation rule F is* monotonic *if for every profile P, agent i, and good g, if $F(P)$ gives g to i, then for every profile P' resulting from P by agent i ranking g higher, leaving everything else (i.e., the relative ranks of all other objects in i's ranking and the rankings of all other agents) unchanged, it holds that $F(P')$ gives g to i. F is* globally monotonic *if for every profile P, agent i, and good g, if $F(P)$ gives g to i, then for every profile P' resulting from P by agent i ranking g higher, all else being equal, we have $F(P')_i = F(P)_i$.*

Clearly, global monotonicity implies monotonicity. These definitions extend to correspondences, but not in a unique way; therefore, we do not consider these extensions in the paper.

Theorem 6 *$F^T_{s,\star}$ is monotonic for every scoring vector s and aggregation function $\star$ (and tie-breaking priority T).*

The proof of Theorem 6 does not establish global monotonicity of $F^T_{s,\star}$; indeed, $\pi = F^T_{s,\star}(P)$ does not imply $\pi = F^T_{s,\star}(P')$ in general. We have the following result.

Proposition 7 *Let T be a separable tie-breaking priority. For each $m \geq 3$ and for each strictly decreasing scoring vector $s = (s_1, \dots, s_m)$, allocation rule $F^T_{s,+}$ is not globally monotonic.*

In order to show that $F^T_{s,\min}$ and $F^T_{s,\text{leximin}}$ do not satisfy global monotonicity, the approach of computing a winning allocation and showing that this allocation is not optimal for the modified profile seems to fail. This is related to the fact that winner determination for both problems is hard and "simple" profiles with (nearly) unique winning allocations do not seem to serve well as counterexamples. Instead, we apply a utility-bounding approach.

Theorem 8 *For each $m \geq 7$ and for each strictly decreasing scoring vector $s = (s_1, \ldots, s_m)$ satisfying, $s_1 - s_2 + s_3 - s_4 > s_m$, allocation rules $F^T_{s,\min}$ and $F^T_{s,\mathrm{leximin}}$ do not satisfy global monotonicity.*

For the remaining cases we conjecture that global monotonicity is not satisfied. This may depend on the tie-breaking mechanism.

Corollary 9 *For each scoring vector $s \in \{\mathrm{borda}, \mathrm{lex}\}$ for $m \geq 7$ goods, allocation rules $F^T_{s,\min}$ and $F^T_{s,\mathrm{leximin}}$ do not satisfy global monotonicity. In addition, for each extended scoring vector s satisfying $s_1(m) > s_2(m) > \cdots > s_m(m)$ for even $m \geq 4$, allocation rules $F^T_{\text{s-qi},\min}$ and $F^T_{\text{s-qi},\mathrm{leximin}}$ do not satisfy global monotonicity either.*

3.3 Object and Duplication Monotonicity and Cloning

Object monotonicity is a dynamic property where additional goods are to be distributed. This means that when new objects are added, no agent is worse off afterwards. In order to define this notion, since some properties need comparability of bundles of goods, we lift agent i's linear order $>_i$ to a strict partial order $\succ_i$ over 2^G by requiring monotonicity ($A \supset B \implies A \succ_i B$) and pairwise dominance (for all $A \subseteq G \setminus \{x,y\}$, $A \cup \{x\} \succ_i A \cup \{y\}$ if $x >_i y$). For strict partial orders we then follow the approach taken by Brams and King [8], Brams, Edelman, and Fishburn [6], and Bouveret, Endriss, and Lang [3]: We distinguish between properties holding *possibly* (i.e., for *some* completion of the partial preferences) and *necessarily* (i.e., for *all* completions).

Definition 10 *Let $\succ$ be a strict partial order over 2^G. We say A is* possibly preferred *to B, $A \succ^{\mathrm{pos}} B$, if there exists a linear order $\succ^*$ refining $\succ$ such that $A \succ^* B$. Analogously, A is* necessarily preferred to *B, $A \succ^{\mathrm{nec}} B$, if for all linear orders $\succ^*$ refining $\succ$ we have $A \succ^* B$. Allowing indifference, we extend $\succ^{\mathrm{pos}}$ to $\succeq^{\mathrm{pos}}$ and $\succ^{\mathrm{nec}}$ to $\succeq^{\mathrm{nec}}$.*

Now, we are ready to define possible and necessary object monotonicity. *These properties are defined for deterministic rules only.*

Definition 11 *Let $P = (>_1, \ldots, >_n)$ be a profile over the set G of goods and let $P' = (>'_1, \ldots, >'_n)$ be a profile that is obtained by adding one more good g to the set of goods, and such that the restriction of P' to G is P. An allocation rule F satisfies* possible (respectively, necessary) object monotonicity *if for all P over G, P' such that P is the restriction of P' over G, and all i, we have $F(P')_i \succeq^{\mathrm{pos}}_i F(P)_i$ (respectively, $F(P')_i \succeq^{\mathrm{nec}}_i F(P)_i$).*

Proposition 12 *For all tie-breaking priorities T, $F^T_{s,+}$ satisfies possible object monotonicity for all scoring vectors s for $n = 2$ agents, yet does not do so for all $n \geq 3$ and strictly decreasing scoring vectors s.*

Necessary object monotonicity might not be true even with only two agents for $F^T_{+,s}$ for some tie-breaking mechanism T. This can be shown by a counterexample (omitted due to lack of space).

Monotonicity in agents has a natural translation in terms of voting power: to give more voting power to a voter, one can just allow her to vote twice (or more). In other words: duplicating a voter will give more weight to her ballot, and give her a higher chance to be heard. For weighted voting games, the related issue of merging and splitting players (a.k.a. false-name manipulation) has been studied [1, 18]. This property has a natural translation to the resource allocation context: informally, two agents having the same preferences will get a better share together than if they were only one participating in the allocation process. More formally:

Definition 13 *Let $P = (>_1, \ldots, >_n)$ be a profile over G and $P' = (>_1, \ldots, >_n, >_{n+1})$ be its extension to $n+1$ agents, where $>_{n+1} = >_n$. An allocation rule F satisfies* possible duplication monotonicity *if $F(P')_n \cup F(P')_{n+1} \succeq^{\mathrm{pos}}_i F(P)_n$; and it satisfies* necessary duplication monotonicity *if $F(P')_n \cup F(P')_{n+1} \succeq^{\mathrm{nec}}_i F(P)_n$.*

It turns out that several scoring allocation rules satisfy at least possible duplication, provided that we use "duplication-compatible" tie-breaking rules, namely, rules T that satisfy the following property: let π and π' be two allocations on $(>_1, \ldots, >_n, >_{n+1})$ (n and $n+1$ being a duplicated agent as above); then $\pi >_T^{n+1} \pi' \Rightarrow (\pi_1, \ldots, \pi_n \cup \pi_{n+1}) >_T^n (\pi'_1, \ldots, \pi'_n \cup \pi'_{n+1})$. For such tie-breaking rules we have:

Theorem 14 *For each scoring vector s, $F_{s,+}$ satisfies possible and necessary duplication monotonicity, and $F_{\text{s-qi,leximin}}$ and $F_{\text{lex,leximin}}$ both satisfy possible duplication monotonicity.*

False-name manipulation has been studied in voting [11, 22], cooperative game theory [1, 18], pseudonymous bidding in combinatorial auctions [23], and, somewhat relatedly, cloning has been studied in voting [21, 13]. Applying this setting to resource allocation, we now assume that agents can participate with multiple identities at the same time. Each of an agent's clones will have the same preferences as this agent. As they are *from the point of view of the agents*, we assume that each agent knows its own linear order over 2^G.

Definition 15 *Let $P = (>_1, \ldots, >_n)$ be a profile of linear orders over G and $\succ_i$ agent i's linear order over 2^G extending $>_i$. An allocation rule F is* susceptible to cloning of agents at P by agent i with $\succ_i$ *if there exists a nonempty set C_i of clones of i (each with the same linear order $>_i$) such that $\bigcup_{j \in C_i \cup \{i\}} \pi'_j \succ_i \pi_i$, where $\pi = (\pi_1, \ldots, \pi_n) = F(P)$, P' is the extension of P to the clones in C_i, and $\pi' = (\pi'_1, \ldots, \pi'_{n+\|C_i\|}) = F(P')$.*

Proposition 16 *If $m \geq 4$ and $m > n$, then for each strictly decreasing scoring vector $s = (s_1, \ldots, s_m)$, allocation rules $F^T_{s,\min}$ and $F^T_{s,\text{leximin}}$ are susceptible to cloning.*

3.4 Consistency and Compatibility

Our scoring allocation rules are based on the maximization of a collective utility defined as the aggregation of individual utilities. An orthogonal classical approach is to find an allocation that satisfies a given (Boolean) criterion. Among the classical criteria, *envy-freeness* states that no agent would be better off with the share of another agent than it is with its own share, and a *Pareto-efficient* allocation cannot be strictly improved for at least one agent without making another agent worse-off.

A natural question is to determine to which extent the scoring allocation rules are compatible with these criteria. More formally:

Definition 17 *Let P be a profile and let X be a property on allocations. An allocation correspondence F is X-*consistent *(respectively, X-*compatible*) if it holds that if there exists an allocation satisfying X for P, then all allocations in $F(P)$ satisfy X (respectively, there is an allocation in $F(P)$ that satisfies X).*

The interpretation is as follows: if F is X-consistent, then no matter which tie-breaking rule is used, an allocation satisfying X will always be found by the allocation rule if such an allocation exists. If F is X-compatible, it means that a tie-breaking rule which is consistent with X (that is: if $\pi \models X$ and $\pi' \not\models X$ then $\pi >_T \pi'$) is needed to find for sure an allocation satisfying X when there is one. Obviously, any X-consistent rule is also X-compatible.

We will now investigate the compatibility and consistency of the scoring rules for Pareto efficiency and envy-freeness. However, these two criteria, which are initially defined for *complete* preorders on 2^G, need to be adapted to deal with *incomplete* preferences.[4] For that, we borrow the following adaptation from [4]. First, given a linear order $\succ$ on G, we say that a mapping $w : G \to R^+$ is *compatible* with $\succ$ if for all $g, g' \in G$, we have $g \succ g'$ if and only of $w(g) > w(g')$; next, given $A, B \subseteq G$, we say that $A \succeq^{pos} B$ if $\sum_{g \in A} w(g) \geq \sum_{g \in B} w(g)$ for some w compatible with $\succ$, and that $A \succeq^{nec} B$ if $\sum_{g \in A} w(g) \geq \sum_{g \in B} w(g)$ for all w compatible with $\succ$. Then:

[4] Recall that we only know the preferences on *singletons* of objects, which have to be lifted to 2^G for the raw criteria to be directly applicable.

Definition 18 *Let $(\succ_1, \ldots, \succ_n)$ be a profile of strict partial orders over 2^G and let π, π' be two allocations. We say (1) π'* possibly Pareto-dominates *π if $\pi'_i \succeq_i^{\text{pos}} \pi_i$ for all i and $\pi'_j \succ_j^{\text{pos}} \pi_j$ for some j; (2) π'* necessarily Pareto-dominates *π if for all $\pi'_i \succeq_i^{\text{nec}} \pi_i$ for all i and $\pi'_j \succ_j^{\text{nec}} \pi_j$ for some j; (3) π is* possibly Pareto-efficient (PPE) *if there is no allocation π' that necessarily Pareto-dominates π; (4) π is* necessarily Pareto-efficient (NPE) *if there is no allocation π' that possibly Pareto-dominates π; (5) π is* possibly envy-free (PEF) *if for every i and j, $\pi_i \succeq_i^{\text{pos}} \pi_j$; (6) π is* necessarily envy-free (NEF) *if for every i and j, $\pi_i \succeq_i^{\text{nec}} \pi_j$.*[5]

An important question is, given a profile P, whether or not there exist a scoring vector s and an aggregation function $\star$ such that the allocation correspondence $F_{s,\star}$ is X-consistent or X-compatible, where $X \in \{\text{NEF, NPE}\}$. While this question is not answered yet in general, we can first observe that $F_{s,+}$ is not NEF-consistent for strictly decreasing scoring vectors. We can also prove that these properties cannot be guaranteed for some of the specific scoring vectors considered here with min or leximin aggregation. Note that if $F_{s,\star}$ is not X-compatible then it is not X-consistent, but the converse is not always true.

Proposition 19 *Let $\star \in \{\min, \text{leximin}\}$. (1) $F_{\text{lex},\star}$ is neither NEF-compatible nor NPE-compatible. (2) $F_{s,\star}$ is neither NEF-consistent nor NPE-compatible for $s \in \{\text{borda}, \text{borda-qi}\}$. (3) $F_{k\text{-app},\star}$ is neither NEF-consistent nor NPE-consistent.*

Proposition 20 *If $n = m$, for each scoring vector s, $F_{s,\min}$ and $F_{s,\text{leximin}}$ are NEF-compatible (and even NEF-consistent for strictly decreasing s) and NPE-compatible.*

4 Winner Determination

In this section, we study the question: What is the complexity of determining an optimal allocation for a given scoring vector and a given aggregation function? For a given scoring vector s and a given aggregation function $F_{s,\star}$, where $\star \in \{+, \min, \text{leximin}\}$, define the following problem concerning winner determination.

$F_{s,\star}$-OPTIMAL-ALLOCATION ($F_{s,\star}$-OA)	
Given:	A profile P of n agents' rankings on a set G of indivisible goods and an allocation π of G.
Question:	Is π in $F_{s,\star}(P)$?

It is easy to see that $F_{s,+}$-OA is in P and both $F_{s,\min}$-OA and $F_{s,\text{leximin}}$-OA are in coNP for every scoring vector s.

The search problem $F_{s,\star}$-FIND-OPTIMAL-ALLOCATION ($F_{s,\star}$-FOA) seeks to actually *find* an optimal allocation. Clearly, $F_{s,+}$-FOA is solvable in polynomial time for any scoring vector s: every good is simply given to an agent who ranks it best. $F_{s,\min}$-FOA and $F_{s,\text{leximin}}$-FOA are much less easy in general.[6] We have the following easy polynomial-time upper bounds for restricted variants.

Proposition 21 *(1) For each k, $F_{k\text{-app},\min}$-FOA is solvable in polynomial time. (2) $F_{s,\min}$-FOA and $F_{s,\text{leximin}}$-FOA are solvable in polynomial time for every scoring vector s if there are a constant number of goods.*

[5]For $i \neq j$, $\pi_i \succeq_i^{\text{pos}} \pi_j$ and $\pi_i \succ_i^{\text{pos}} \pi_j$ ($\pi_i \succeq_i^{\text{nec}} \pi_j$ and $\pi_i \succ_i^{\text{nec}} \pi_j$) are equivalent, as the bundles to be compared are always disjoint.

[6]Clearly, if the scoring vector *s is part of the input* then the problem $F_{s,\star}$-FOA is (weakly) NP-hard, even for two agents having the same preferences, by a direct reduction from PARTITION.

	$F^T_{s,+}$	$F^T_{s,\min}$	$F^T_{s,\text{leximin}}$
separability	✗[1]	✗[1]	✗[1]
monotonicity	✓	✓	✓
global monotonicity	✗[1,2]	✗[1,3]	✗[1,3]
pos. object mon.	✓($n = 2$), ✗($n \geq 3$)		
nes. object mon.	✗[4]		
pos. duplication mon.	✓[5]		✓[5]
nes. duplication mon.	✓[5]		
susceptible to cloning		✓[1] ($m > n$)	✓[1] ($m > n$)
PEF			
-compatible		✓($m = n$)	✓($m = n$)
-consistent		✓[1] ($m = n$)	✓[1] ($m = n$)
NEF			
-compatible		✗[8], ✓($m = n$)	✗[8], ✓($m = n$)
-consistent	✗[1]	✗[6],✗[7],✗[8], ✓[1] ($m = n$)	✗[6],✗[7],✗[8], ✓[1] ($m = n$)
PPE-			
-compatible		✓($m = n$)	✓($m = n$)
-consistent			
NPE			
-compatible		✗[6],✗[8], ✓($m = n$)	✗[6],✗[8], ✓($m = n$)
-consistent		✗[6],✗[7],✗[8]	✗[6],✗[7],✗[8]

[1] for strictly decreasing scoring vector
[2] for separable tie-breaking T
[3] additional restrictions on the scoring vector
[4] depends on the tie-breaking T
[5] for duplication-compatible tie-breaking T
[6] for $s \in \{\text{borda}, \text{borda-qi}\}$
[7] for $s = k$-app
[8] for $s = \text{lex}$

Table 1: Overview of axiomatic results

(1) is a special case of the problem of maximizing egalitarian social welfare with a $\{0,1\}$-additive function, known to be solvable in polynomial time by applying a network flow algorithm [14]. In addition, we will study the following decision problem associated with the value of an optimal allocation.

$F_{s,+}$-OPTIMAL-ALLOCATION-VALUE ($F_{s,+}$-OAV)	
Given:	A profile $P = (>_1, \ldots, >_n)$ of n agents' rankings on a set G of indivisible goods and $k \in \mathbb{N}$.
Question:	Is there an allocation $\pi = (\pi_1, \ldots, \pi_n)$ such that $\sum_{1 \leq i \leq n} u_{>_i,s}(\pi_i) \geq k$?

Analogously, we define $F_{s,\min}$-OAV by asking whether or not $\min_{1 \leq i \leq n} u_{>_i,s}(\pi_i) \geq k$, and $F_{s,\text{leximin}}$-OAV where the bound is an ordered list $(k_1, \ldots, k_n)$ of nonnegative integers and we ask whether $(u_{>_1,s}(\pi_1), \ldots, u_{>_n,s}(\pi_n)) \geq_{\text{leximin}} (k_1, \ldots, k_n)$. Table 2 summarizes our complexity results.

Clearly, $F_{s,+}$-OAV is in P. Since the value of a given allocation for min and leximin can be computed in polynomial time, $F_{s,\min}$-OAV and $F_{s,\text{leximin}}$-OAV are in NP for each scoring rule s. For lexicographic scoring and quasi-indifference, these bounds are tight.

	OA	OAV	FOA
$F_{s,+}$	in P	in P	pol. time
$F_{s,\min}$	coNP-comp*	NP-comp*	NP-hard*
k-app or $m \in O(1)$	in P	in P	pol. time
lex or e-qi	coNP-comp$^\#$	NP-comp$^\#$	NP-hard$^\#$
borda	coNP-comp	NP-comp	
lex or borda or e-qi, if $n \in O(1)$	in P	in P	pol. time
$F_{s,\text{leximin}}$	coNP-comp*	NP-comp*	NP-hard*
lex or e-qi	in coNP	NP-comp$^\#$	NP-hard$^\#$
borda	in coNP	NP-comp	
lex or borda or e-qi, if $n \in O(1)$	in P	in P	pol. time

*if s is part of the input (even for two agents with same preferences)

$^\#$ where e is a strictly decreasing scoring vector

Table 2: Overview of complexity results (gray: partial results)

Theorem 22 $F_{\text{lex},\min}$-OAV *and* $F_{\text{lex},\text{leximin}}$-OAV *both are* NP-*complete.*

Theorem 23 *For each fixed and strictly decreasing scoring vector e,* $F_{e\text{-qi},\min}$-OAV *and* $F_{e\text{-qi},\text{leximin}}$-OAV *both are* NP-*complete.*

An anonymous reviewer of a previous draft of this paper obtained the following result.

Theorem 24 $F_{\text{borda},\min}$-OAV *and* $F_{\text{borda},\text{leximin}}$-OAV *both are* NP-*complete.*

Using a slight adaptation of the proofs of Theorems 22, 23 and 24, we can show that $F_{\text{lex},\min}$-OA, $F_{\varepsilon\text{-qi},\min}$-OA and $F_{\text{borda},\min}$-OA are coNP-complete. These proofs, however, do not directly extend to the problems $F_{\text{lex},\text{leximin}}$-OA, $F_{\varepsilon\text{-qi},\text{leximin}}$-OA nor $F_{\text{borda},\text{leximin}}$-OA.

Proposition 25 *For each fixed and strictly decreasing scoring vector e, for each* $s \in \{\text{borda}, \text{lex}, e\text{-qi}\}$*,* $F_{s,\min}$-OA *is* coNP-*complete.*

For a constant number of agents, we provide efficient algorithms for many of our problems via dynamic programming.

Theorem 26 *For each scoring vector e with polynomial (in m) bounded entries, for each* $s \in \{\text{borda}, \text{lex}, e\text{-qi}\}$ *and for each* $\star \in \{\min, \text{leximin}\}$*,* $F_{s,\star}$-OA *and* $F_{s,\star}$-FOA *are solvable in polynomial time if the number of agents is constant.*

5 Approximation

$F_{\text{lex},\min}$-OAV is NP-complete by Theorem 22. This raises the issue of whether there exists a polynomial-time approximation algorithm for the search variant of this rule; this turns out to be the case.

Proposition 27 *There exists a* $(1/2)$*-approximation algorithm for* $F_{\text{lex},\min}$-FOA.

We now turn to a different kind of approximation: *picking sequences*, whose advantage is that they avoid preference elicitation. We investigate the price to pay for that: in Section 5.1 (respectively, Section 5.2), we focus on the ratio (respectively, the difference) between the value of the optimal allocation and the value of the allocation obtained by applying a picking sequence.

5.1 Multiplicative Price of Elicitation-Freeness

Simple protocols for allocating indivisible resources without eliciting the agents' preferences first, as discussed in [9, 5, 16], consist in asking agents to pick objects one after the other, following a predefined sequence. An interesting question is whether using such protocols (without elicitation), or simulating them from the known preferences (after full elicitation of the agents' rankings) gives a good approximation of our scoring rules: what is the loss incurred by the application (simulated or not) of the picking sequence with respect to an optimal allocation? We give here two results for Borda scoring: one for egalitarianism, one for utilitarianism. One may wonder why we should look for such a result in the case of utilitarianism, given that there is a straightforward greedy algorithm that outputs an optimal allocation. The reason is that picking sequences (when actually used, as opposed to simulated ones) do better on one criterion: they are very cheap in communication, as agents only reveal part of their preferences by picking objects, as opposed to revealing their full preferences in the case of a centralized protocol.

Formally, a *(picking) policy* is a sequence $\sigma = \sigma_1 \cdots \sigma_m \in \{1,\ldots,n\}^m$, where at each step, agent σ_i picks her most preferred object among those remaining (where we assume agents to use only their sincere picking strategies). For instance, if $m = 4$ and $n = 2$, 1221 is the sequence where 1 picks an object first, then 2 picks two objects, and 1 takes the last object. The precise definition of an allocation induced by a picking sequence and a profile, assuming that agents act according to their true preferences, is in [5]. Sequential allocation rules are appealing because they require even less input from the agents than singleton-based allocation rules; however, this gain in communication comes with a loss of social welfare. To quantify this loss, we define the following measure.

Definition 28 *Given a policy σ (for n agents and m objects), a scoring vector s, and an aggregation function $\star \in \{+,\min\}$, the* multiplicative price of elicitation-freeness *of σ, which we denote by $MPEF_{s,\star}(\sigma)$, is the worst-case ratio in social welfare between an optimal allocation for $F_{s,\star}$ and the sequential allocation, among all profiles with m goods.*

Since we focus on $s =$ borda only, we from now on simply write $MPEF_\star(\sigma)$ to mean $MPEF_{\text{borda},\star}(\sigma)$. We now give results about the quality of the outcome of *balanced picking sequences* $(12\cdots n)^{\frac{m}{n}}$, assuming that m is a multiple of n. For instance, if $m = 6$ and $n = 3$, $\sigma = 123123$ is balanced.

Computing the price of elication-freeness is challenging. We focus on the *regular policy* $\sigma_{\mathrm{R}}^n = (1\cdots n)^*$, but our results are very similar to those for other fair policies such as $(1\cdots nn\cdots 1)^*$.

5.1.1 Lower Bounds

A naive algorithm for computing the additive or multiplicative PEF for a given value m is simply to generate all possible profiles and for each of them to compute an optimal allocation from which it is possible to deduce the loss incurred by the sequential allocation. However, the number of profiles grows exponentially in m, and computing an optimal allocation might be intractable. Still, it is possible to lower-bound the PEF for a given m by computing the incurred loss for a subset of all possible profiles. The conclusions that can be drawn from computational experiments is that for $\star = +$, in the worst case the loss seems to be in the order of m (which is good), whereas in the average case the loss seems to grow also linearly with m. The conclusions for $\star = \min$ are somewhat similar, but they are less firm, as we have not been able to go as far in the number of objects as for $\star = +$. We now provide a formal lower bound for *MPEF* for $\star = +$, and the regular policy.

Proposition 29 *For $m = kn$ objects, $MPEF_+(\sigma_R^n) \geq 1 + \frac{mn-m-n^2+n}{m^2+mn}$, and thus we have $MPEF_+(\sigma_R^n) \geq 1 + \frac{n-1}{m} + \Theta(1/m^2)$ when m tends to $+\infty$ with n being held constant.*

5.1.2 Upper Bounds

We now also provide formal upper bounds for *MPEF* for $\star = +$ and $\star = \min$, and the regular policy.

Proposition 30 *For $m = kn$ objects, $MPEF_+(\sigma_R^n) \leq 2 - \frac{m-n}{mn+n}$, and thus $MPEF_+(\sigma_R^n) \leq 2 - \frac{1}{n} + \Theta(1/m)$ when m tends to $+\infty$ with n being held constant.*

Corollary 31 *If $n = 2$ and $m = 2k$, $1 + \frac{m-2}{m(m+2)} \leq MPEF_+(\sigma_R^2) \leq \frac{3}{2} + \frac{3}{2m+2}$.*

Proposition 32 *For $m = kn$ objects, $MPEF_{\min}(\sigma_R^n) \leq \frac{2mn-m+n}{mn+2n-n^2}$, and thus $MPEF_+(\sigma_R^n) \leq 2 - \frac{1}{n} + \Theta(1/m)$ when m tends to $+\infty$ with n being held constant.*

Corollary 33 *If $n = 2$ and $m = 2k$, $MPEF_{min}(\sigma_R^2) \leq \frac{3}{2} + \frac{5}{m+4}$.*

5.2 Additive Price of Elicitation-Freeness

Definition 34 *Given a policy σ (for n agents and m objects), a scoring vector s, and an aggregation function $\star \in \{+, \min\}$, the* additive price of elicitation-freeness *of σ, denoted by $APEF_{s,\star}(\sigma)$, is the worst-case difference in social welfare between the sequential allocation and an optimal allocation for $F_{s,\star}$ among all profiles with m goods.*

Since we focus on $s =$ borda only, we simply write $APEF_\star(\sigma)$ to mean $APEF_{\text{borda},\star}(\sigma)$.

We now provide a formal lower bound linear in m for $\star = +$, with a fixed number of agents n and the regular policy.

Proposition 35 *For $m = kn$ objects, $APEF_+(\sigma_R^n) \geq \frac{(n-1)(m-n)}{2}$.*

We now also provide a formal upper bound quadratic in m with a fixed number of agents n, for $\star = +$ and $\star = \min$, and the regular policy.

Proposition 36 *For $m = kn$ objects, $APEF_+(\sigma_R^n) \leq \frac{(m-n)(mn-m+n^2+n)}{2n}$.*

Corollary 37 *For $n = 2$ and $m = 2k$, $\frac{m}{2} - 1 \leq APEF_+(\sigma_R^2) \leq \frac{m^2}{4} + m - 3$.*

Proposition 38 *For $m = kn$ objects, $APEF_{\min}(\sigma_R^n) \leq \frac{m^2n-mn-m^2+mn^2}{2n^2}$.*

This upper bound is asymptotically better (by a factor of n) than the upper bound for $APEF_+(\sigma_R^n)$. In particular, for two agents, it is in the order of $m^2/8$ (to be compared with $m^2/4$ for $\star = +$ in Corollary 37).

6 Concluding Remarks

Generalizing earlier work [8, 6], we have defined a family of rules for the allocation of indivisible goods to agents that are parameterized by a scoring vector and an aggregation function. We have discussed a few key properties, and for each of them we have given some positive as well as some negative results about their satisfaction by scoring allocation rules. The relatively high number of negative results should be balanced against the satisfaction of several important properties (including monotonicity) together with the simplicity of these rules. And anyway, defining allocation rules of indivisible goods from ordinal inputs on other principles does not look easy at all. Our results on axiomatic properties are far from being complete: for many properties we do not have an exact characterization of the scoring allocation rules that satisfy them, and obtaining such exact characterizations is left for further research.

In addition, focusing on four important scoring vectors and three central aggregation functions, we have determined the complexity of computing an optimal allocation for almost all rules considered here (see Table 2 for the list of results, and the problems whose precise complexity remains unknown). We have also given some approximation results, some of which make use of picking sequences whose main purpose it is to avoid preference elicitation.

Even if winner determination is computationally difficult for many choices of s and $\star$ (except for the trivial case of $\star = +$), these rather negative results should be tempered by the fact that in most practical settings the number of agents and items is sufficiently small for the optimal allocation to be computed, even when its determination is NP-hard. Moreover, the results of Section 5 show that good approximations of optimal allocations can often be determined with a very low communication cost. An issue that we did not consider here is manipulability. Almost all of our rules seem to be manipulable; characterizing exactly the family of allocation rules that are manipulable and measuring the extent to which our rules are computationally resistant to manipulation is clearly an interesting topic for further research.

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