Une courte introduction au choix social
Réformons l'élection législative !

Sylvain Bouveret
LIG – Univ. Grenoble-Alpes

CED Cours transversal Sciences Environnement Sociétés

Grenoble, 31 mai 2021

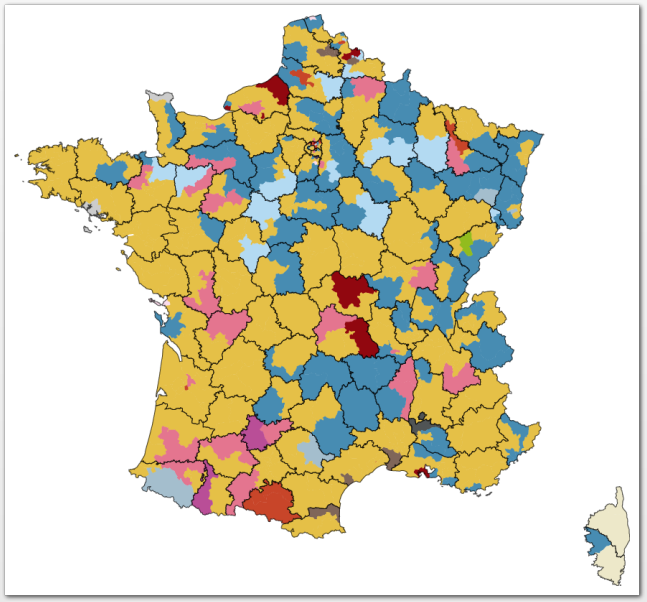
Parliamentary election

Voting simulations for the electoral reform

The National Assembly

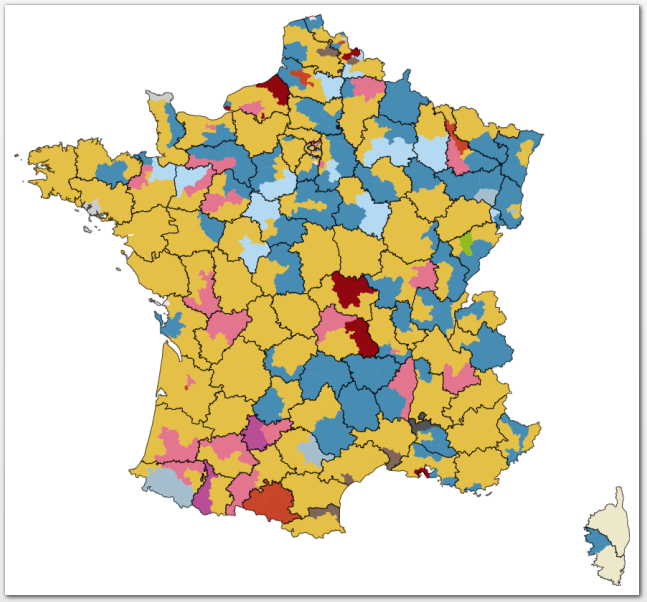
Welcome to the French National Assembly...

The official parliamentary election



- 577 deputies
- Two-round majority by circonscription

The official parliamentary election



- 577 deputies
- Two-round majority by circonscription

- >50% of valid votes and >25% of registered voters ⇒ **elected**
- >12.5% of reg. voters ⇒ 2^{nd} round
- if this rule does not select at least 2 candidates, then the two candidates with highest plurality score go to the 2^{nd} round
- 2^{nd} round: plurality voting

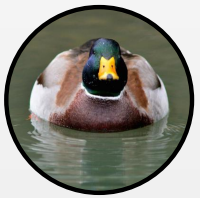
Let's play a game...

5 candidates in a (fictious) district:

Let's play a game...

5 candidates in a (fictious) district:

Ann
Ducks United



Bob
Yes We Canne

Carol
Seed For Everyone

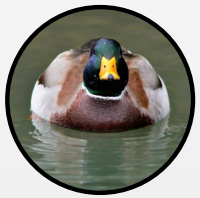
Donald
The Duckublicans

Emma
Chickens Out!

Let's play a game...

5 candidates in a (fictious) district:

Ann
Ducks United



Bob
Yes We Canne

Carol
Seed For Everyone

Donald
The Duckublicans

Emma
Chickens Out!

Who qualifies for the second round?

What the duck?

Quiz : Who qualifies for the second round?

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.
20%	2%	8%	40%	5%	25%

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.
20%	2%	8%	40%	5%	25%

$\rightarrow$ D wins (no 2^{nd} round)

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.
20%	2%	8%	40%	5%	25%

$\rightarrow$ D wins (no 2^{nd} round)

A	B	C	D	E	abst.
2%	2%	13%	21%	2%	60%

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.	
20%	2%	8%	40%	5%	25%	$\rightarrow$ D wins (no 2^{nd} round)

A	B	C	D	E	abst.	
2%	2%	13%	21%	2%	60%	$\rightarrow$ C, D

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.	
20%	2%	8%	40%	5%	25%	$\rightarrow$ D wins (no 2^{nd} round)

A	B	C	D	E	abst.	
2%	2%	13%	21%	2%	60%	$\rightarrow$ C, D

A	B	C	D	E	abst.
13%	13%	14%	15%	20%	25%

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.	
20%	2%	8%	40%	5%	25%	$\rightarrow$ D wins (no 2^{nd} round)

A	B	C	D	E	abst.	
2%	2%	13%	21%	2%	60%	$\rightarrow$ C, D

A	B	C	D	E	abst.	
13%	13%	14%	15%	20%	25%	$\rightarrow$ A, B, C, D, E

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.	
20%	2%	8%	40%	5%	25%	→ D wins (no 2nd round)

A	B	C	D	E	abst.	
2%	2%	13%	21%	2%	60%	→ C, D

A	B	C	D	E	abst.	
13%	13%	14%	15%	20%	25%	→ A, B, C, D, E

A	B	C	D	E	abst.
12%	10%	11%	10%	22%	25%

What the duck?

Quiz : Who qualifies for the second round?

A	B	C	D	E	abst.	
20%	2%	8%	40%	5%	25%	→ D wins (no 2nd round)

A	B	C	D	E	abst.	
2%	2%	13%	21%	2%	60%	→ C, D

A	B	C	D	E	abst.	
13%	13%	14%	15%	20%	25%	→ A, B, C, D, E

A	B	C	D	E	abst.	
12%	10%	11%	10%	22%	25%	→ A, E

Objectives

The story begins with Cédric Villani

- Mathematician (Field medal 2010)
- Deputy (LREM) since 2017

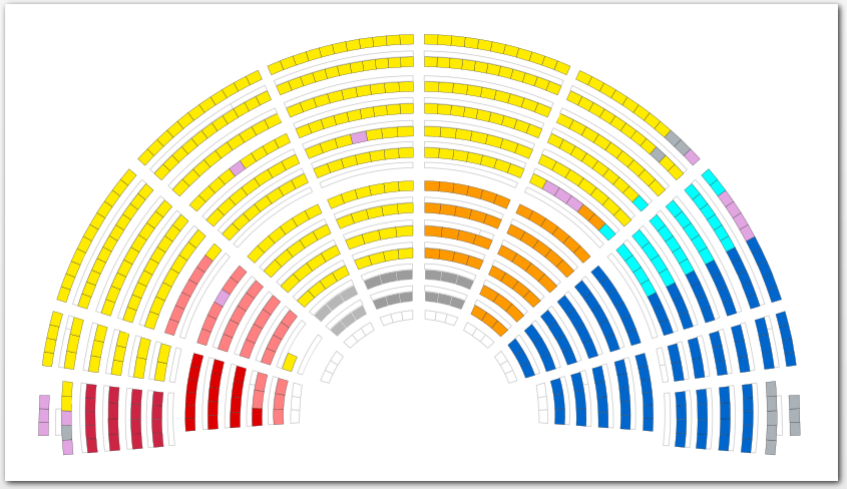
Objectives

The story begins with Cédric Villani

- Mathematician (Field medal 2010)
- Deputy (LREM) since 2017

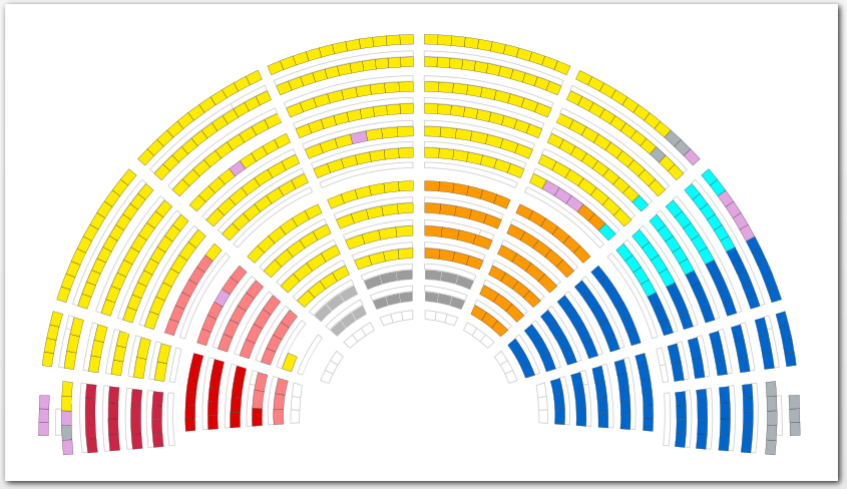
Objectives

This is the current composition of the National Assembly (2017):



Objectives

This is the current composition of the National Assembly (2017):

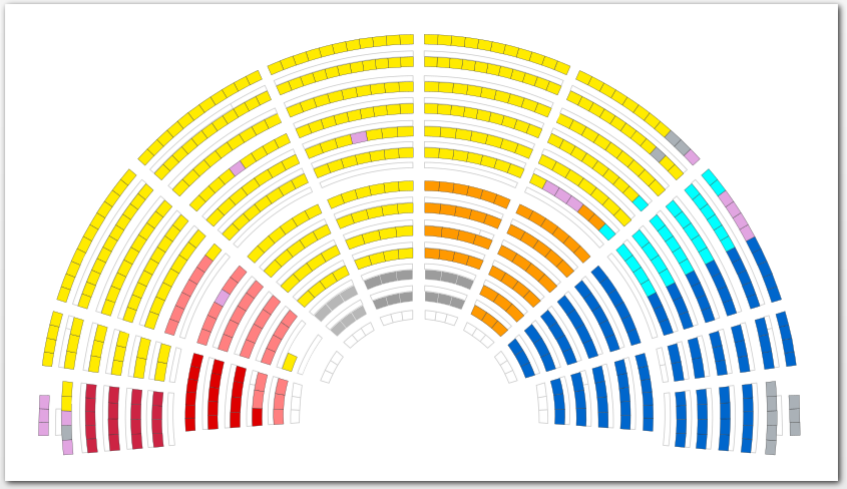


What would have happened if:

- the number of deputies is reduced?
- some amount of proportionality is introduced?

Objectives

This is the current composition of the National Assembly (2017):



What would have happened if:

- the number of deputies is reduced?
- some amount of proportionality is introduced?

Run computer simulations to replay the match. You have 1 month++.

Team

- **Renaud Blanch**, LIG, Université Grenoble-Alpes
- **Sylvain Bouveret**, LIG, Université Grenoble-Alpes

Contributions of:

- **Jérôme Lang**, LAMSADE CNRS, Université Paris-Dauphine
- **Bruno Cautrès**, CNRS, CEVIPOF - Centre de recherches politiques de Sciences Po.

Previous report:

Cohendet, M.-A., Lang, J., Laslier, J.-F., Pech, T., and Sawicki, F. (2018).
Une "dose de proportionnelle" : pourquoi ? comment ? laquelle ?
Technical report, Terra Nova.

Datasets

- Official results (Ministère de l'Intérieur)
  - per circonscription (electoral district)
  - per canton
- Geographic boundaries
  - circonscriptions: Atelier de cartographie de Sciences Po.
  - cantons: IGN
  - départements: IGN Geofla

Parliamentary election

Back to basic: proportionality?

Of parties and districts

- One vote = one district + one party

Voter	District	Party
Voter #1	Circ.1	LREM
Voter #2	Circ.1	FI
Voter #3	Circ.1	LREM
Voter #4	Circ.1	LR
Voter #5	Circ.2	PS
Voter #6	Circ.2	PS
Voter #7	Circ.2	EELV
Voter #8	Circ.3	EELV
⋮		

Majoritarian rule

- Current rule: priority = district. Principle: 1 district = 1 deputy
- May totally ignore the representation of parties

Quiz : Can you imagine a situation where one party wins all the seats in the parliament with only 12.5% of the votes?

Majoritarian rule

- Current rule: priority = district. Principle: 1 district = 1 deputy
- May totally ignore the representation of parties

Quiz : Can you imagine a situation where one party wins all the seats in the parliament with only 12.5% of the votes?

At the other end of the spectrum, full proportionality totally ignores districts (= nation-wise election)

Majoritarian rule

- Current rule: priority = district. Principle: 1 district = 1 deputy
- May totally ignore the representation of parties

Quiz : Can you imagine a situation where one party wins all the seats in the parliament with only 12.5% of the votes?

At the other end of the spectrum, full proportionality totally ignores districts (= nation-wise election)

Mixed voting:

- Mixed voting tries to reconcile majoritarian and proportional vote
- There are a lot of ways to do it

The apportionment problem

Similarities with the **apportionment problem**

Name	Gender	Group	Age	Affiliation
Ann	F	A	J	L
Bob	M	A	J	E
Charlie	M	A	S	L
Donna	F	B	S	E
Ernest	M	A	S	L
George	M	A	S	E
⋮				

How to elect a committee that reflects the diversity of the population?

Example borrowed from [Lang and Skowron, 2018]

Lang, J. and Skowron, P. (2018).
Multi-attribute proportional representation.
Artificial Intelligence, 263:74–106.

Parliamentary election

What did we simulate?

Voting rules

- Current voting rule (two-round majority)
- Mixed rules (majority / proportional): k seats allocated using the current rule / $n - k$ seats allocated proportionally to a vector $(p_1, \ldots, p_n)$ that depends on the method used
  - **Additive**: proportions using raw party scores
  - **Compensatory**: proportions using deputy deficits
  - **Corrective**: proportions using vote deficits
- Mixed rule used for Senate

Three mixed rules

Additive rule

ECO obtains 4.3% of the votes and REM 28.2%

- $p_{ECO} = 4.3\%$
- $p_{REM} = 28.2\%$ (no matter how much majority seats they have)

Three mixed rules

Additive rule

ECO obtains 4.3% of the votes and REM 28.2%

- $p_{ECO} = 4.3\%$
- $p_{REM} = 28.2\%$ (no matter how much majority seats they have)

Compensatory rule

ECO obtains 1 seat, but should obtain 24.811 in a fully proportional election $\Rightarrow$ deficit: 23.811
REM obtains 308 seats, but should obtain 162.714 in a fully proportional election $\Rightarrow$ deficit: -145.286

- $p_{ECO} = 23.811 / \sum p_i$
- $p_{REM} = 0$

Three mixed rules

Corrective rule

FI wins in 17 districts. The (spoiled) FI voters in the other 560 districts are $2,392,951$
REM wins in 308 districts. The (spoiled) FI voters in the other 269 districts are $1,714,010$

- $p_{FI} = 2,392,951/\sum p_i$
- $p_{REM} = 1,714,010/\sum p_i$

About the remainders

The proportions do not yield integral numbers of deputies...

About the remainders

The proportions do not yield integral numbers of deputies...

How to allocate the remainders?

About the remainders

The proportions do not yield integral numbers of deputies...

How to allocate the remainders?

- **d'Hondt method** (highest average): order the parties by decreasing ratios votes / seats and allocate the remaining seats sequentially
- **Hare method** (highest remainder): order the parties by decreasing differences proportion - seats and allocate the remaining seats sequentially

The core difficulties

Reducing the number of majoritarian deputies $\Rightarrow$ reducing the number of electoral districts

The core difficulties

Reducing the number of majoritarian deputies $\Rightarrow$ reducing the number of electoral districts

- **Redistricting**: what will be the new districts?

The core difficulties

Reducing the number of majoritarian deputies $\Rightarrow$ reducing the number of electoral districts

- **Redistricting**: what will be the new districts?
- **2nd round prediction**: who will win the new second rounds???

The core difficulties

Reducing the number of majoritarian deputies $\Rightarrow$ reducing the number of electoral districts

- **Redistricting**: what will be the new districts?
- **2$^{\text{nd}}$ round prediction**: who will win the new second rounds???

Remark: a similar project: *Dérangeons la Chambre* (R. Magni-Berton, 2016)

http://www.derangeonslachambre.fr/

But does not simulate a reduction of the number of districts

Redistricting

Three methods for redistricting:

- **Manual (J. Lang)**: scenario with 404 districts

Redistricting

Three methods for redistricting:

- **Manual (J. Lang)**: scenario with 404 districts
- **Statistical**: random generation of artificial districts with the same statistical parameters than the actual ones

Redistricting

Three methods for redistricting:

- **Manual (J. Lang)**: scenario with 404 districts
- **Statistical**: random generation of artificial districts with the same statistical parameters than the actual ones
- **Geographic merging:**

Redistricting

Three methods for redistricting:

- **Manual (J. Lang)**: scenario with 404 districts
- **Statistical**: random generation of artificial districts with the same statistical parameters than the actual ones
- **Geographic merging:**
  - Uniform reduction of the number of districts per department

Redistricting

Three methods for redistricting:

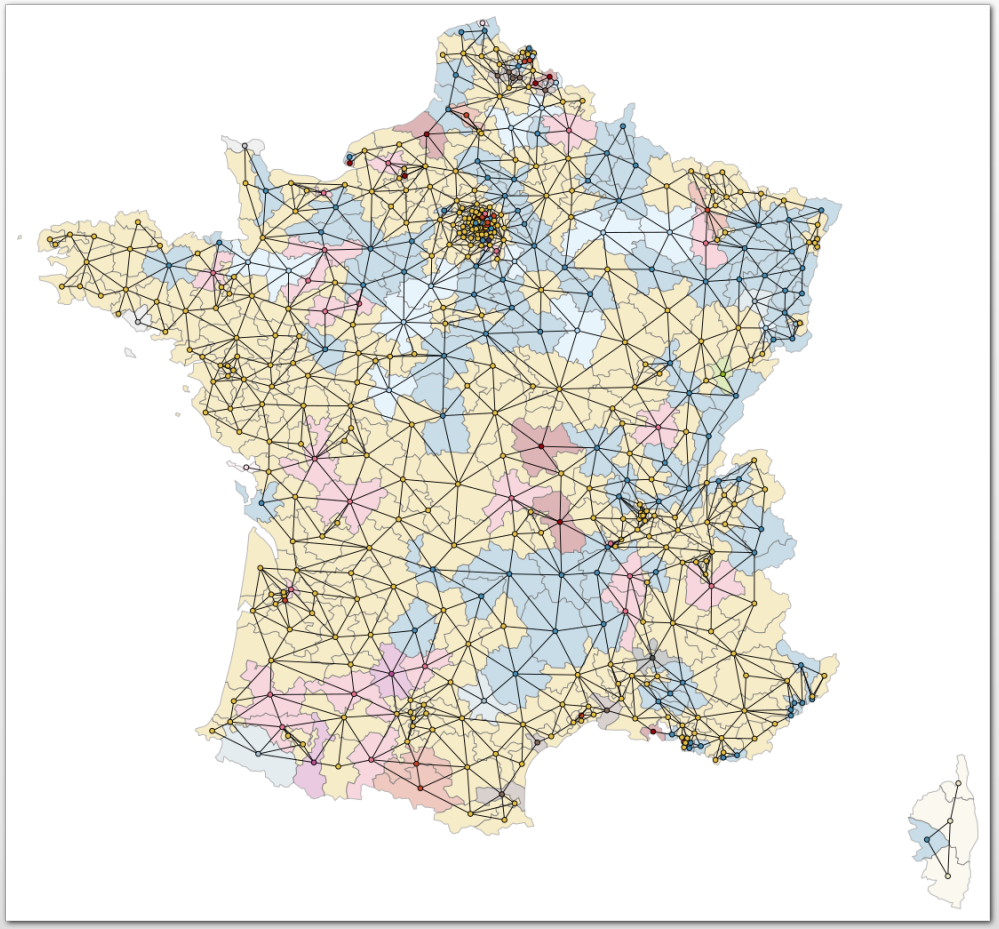
- **Manual (J. Lang)**: scenario with 404 districts
- **Statistical**: random generation of artificial districts with the same statistical parameters than the actual ones
- **Geographic merging:**
  - Uniform reduction of the number of districts per department
  - Uniform merging inside departments, with connectivity constraints (graph partitioning)

Redistricting

Three methods for redistricting:

- **Manual (J. Lang)**: scenario with 404 districts
- **Statistical**: random generation of artificial districts with the same statistical parameters than the actual ones
- **Geographic merging:**
  - Uniform reduction of the number of districts per department
  - Uniform merging inside departments, with connectivity constraints (graph partitioning)
  - Circonscription or canton-based

Redistricting by geographic merging



2^{nd} round prediction

Four methods for 2^{nd} round prediction:

- **Naive**: 2^{nd} round winner $= 1^{st}$ round winner

2^{nd} round prediction

Four methods for 2^{nd} round prediction:

- **Naive**: 2^{nd} round winner $= 1^{st}$ round winner
- **Single-peaked**: votes transfer to the closest candidate

2^{nd} round prediction

Four methods for 2^{nd} round prediction:

- **Naive**: 2^{nd} round winner $= 1^{st}$ round winner
- **Single-peaked**: votes transfer to the closest candidate
- **Empirical**: transfer matrix (B. Cautrès, Cevipof)

2^{nd} round prediction

Four methods for 2^{nd} round prediction:

- **Naive**: 2^{nd} round winner $= 1^{st}$ round winner
- **Single-peaked**: votes transfer to the closest candidate
- **Empirical**: transfer matrix (B. Cautrès, Cevipof)
- **Learning model**: carefully tuned linear predictor

2^{nd} round prediction

Four methods for 2^{nd} round prediction:

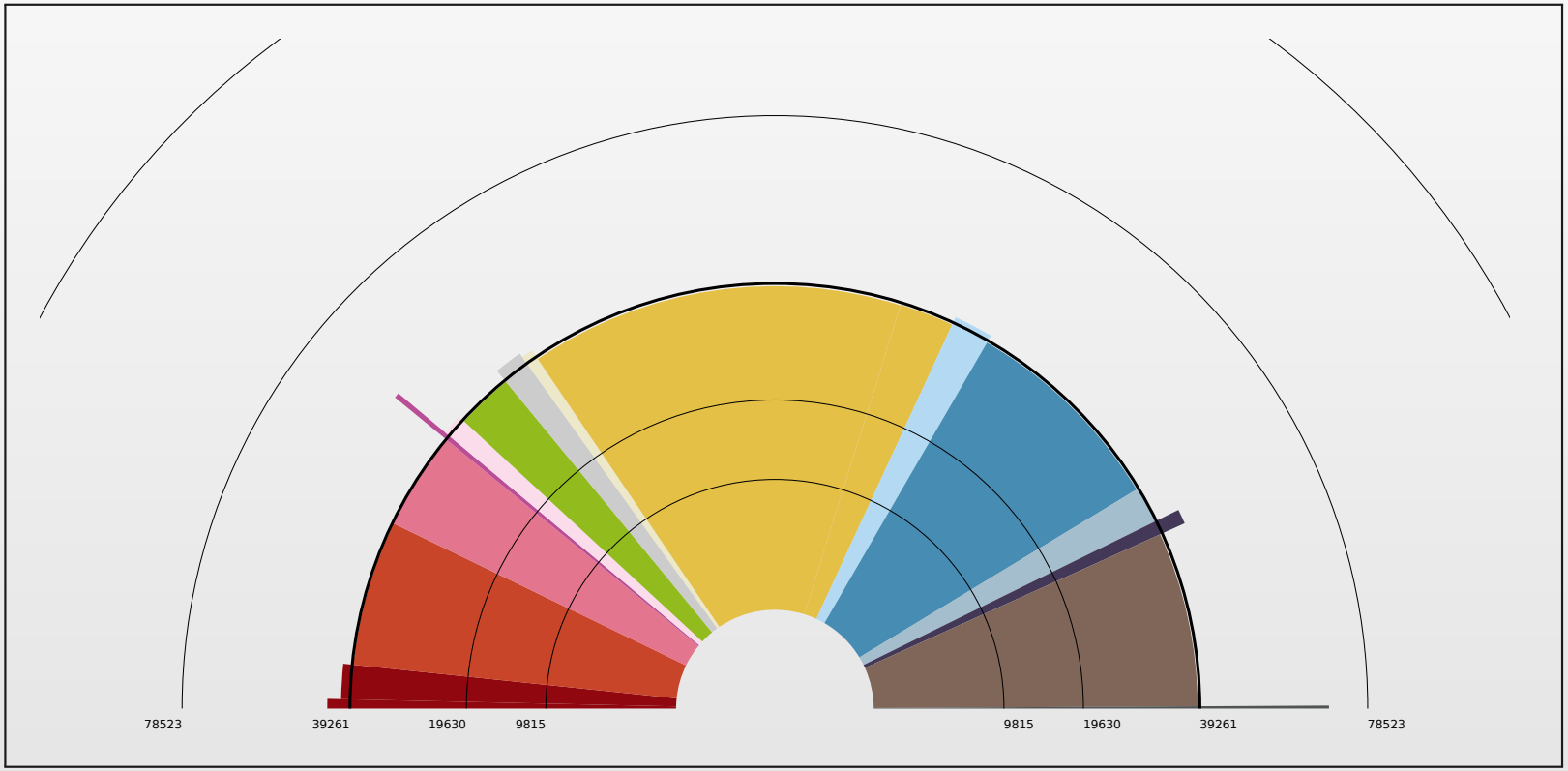
- **Naive**: 2^{nd} round winner $= 1^{st}$ round winner
- **Single-peaked**: votes transfer to the closest candidate
- **Empirical**: transfer matrix (B. Cautrès, Cevipof)
- **Learning model**: carefully tuned linear predictor

Conclusion: the linear predictor seems to give the best results (but the validity of the method is very doubtful...)

Parliamentary election

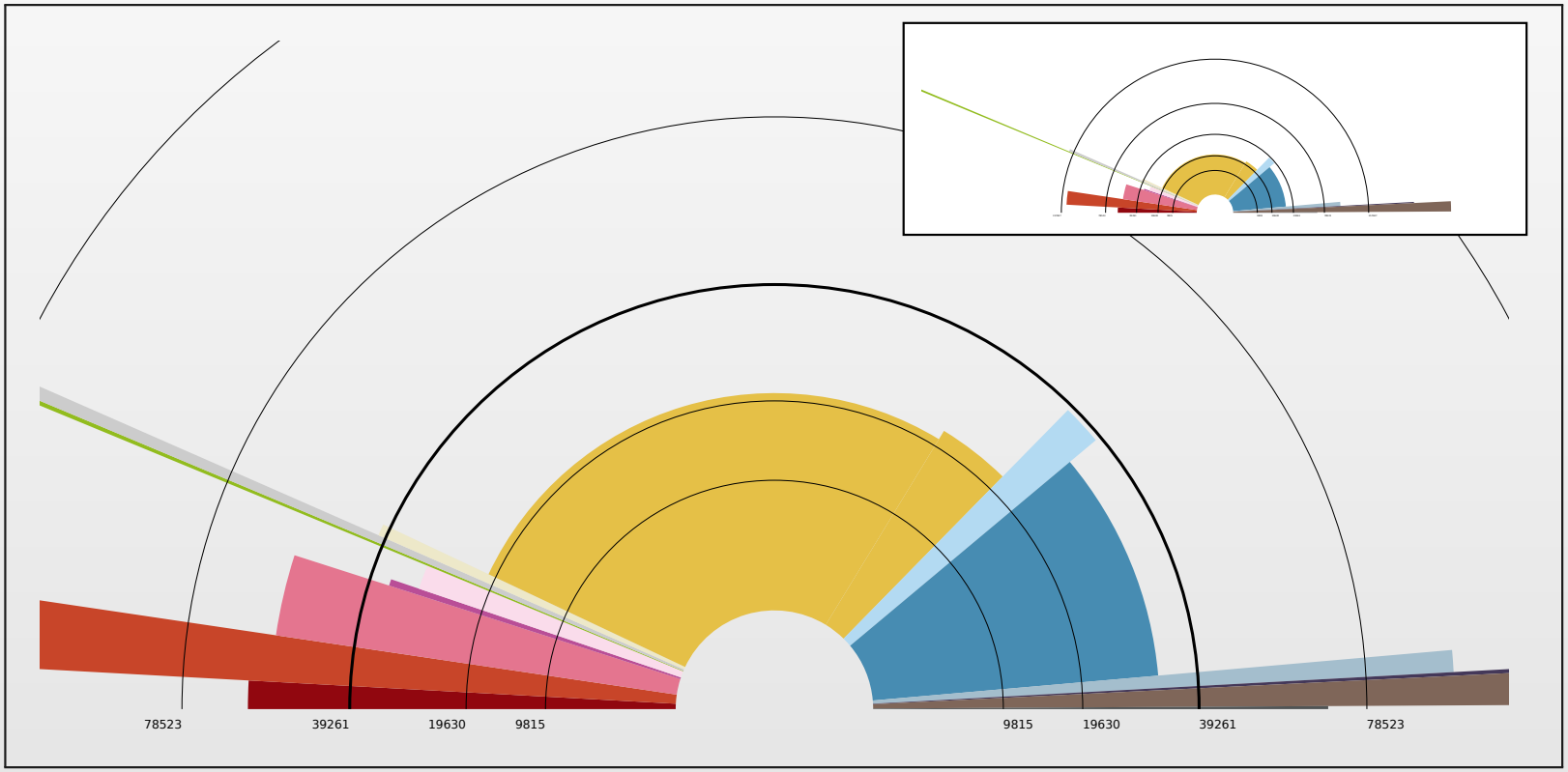
Results of the simulations

Visualizing proportionality



Purely proportional voting rule, 577 deputies

Visualizing proportionality



Purely majoritarian voting rule (current electoral system)

Measuring proportionality

We use a metrics introduced by Loosemore and Hanby (1971):

p measure of proportionality

Sum for each party of the number of voters to add or remove to obtain full proportionality.
$\Leftrightarrow$ sum of the areas above and below the proportionality circle

$q = 1 - p$, so that $1 =$ full proportionality.

Loosemore, J. and Hanby, V. J. (1971).
The Theoretical Limits of Maximum Distortion: Some Analytical Expressions for Electoral Systems.
British Journal of Political Science, 1.

Measuring proportionality

We use a metrics introduced by Loosemore and Hanby (1971):

p measure of proportionality

Sum for each party of the number of voters to add or remove to obtain full proportionality.
$\Leftrightarrow$ sum of the areas above and below the proportionality circle

$q = 1 - p$, so that $1 =$ full proportionality.

Loosemore, J. and Hanby, V. J. (1971).
The Theoretical Limits of Maximum Distortion: Some Analytical Expressions for Electoral Systems.
British Journal of Political Science, 1.

Alternative interpretation: proportion of deputies that stay in place if we have to make the parliament fully proportional.

Measuring proportionality

We use a metrics introduced by Loosemore and Hanby (1971):

p measure of proportionality

Sum for each party of the number of voters to add or remove to obtain full proportionality.
$\Leftrightarrow$ sum of the areas above and below the proportionality circle

$q = 1 - p$, so that $1 =$ full proportionality.

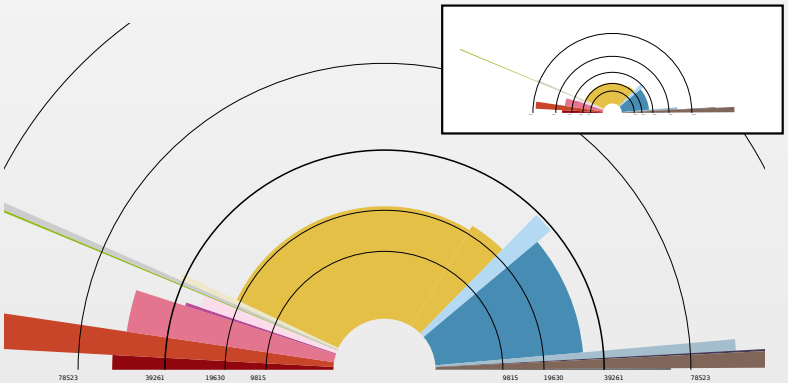
Loosemore, J. and Hanby, V. J. (1971).
The Theoretical Limits of Maximum Distortion: Some Analytical Expressions for Electoral Systems.
British Journal of Political Science, 1.

Alternative interpretation: proportion of deputies that stay in place if we have to make the parliament fully proportional.

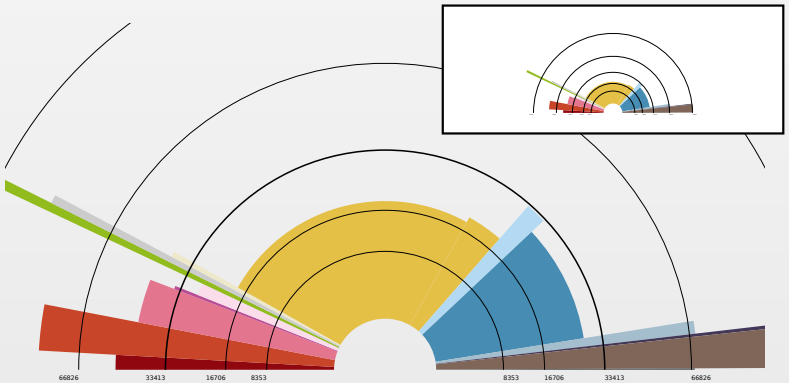
- Purely proportional voting rule, 577 deputies: $q = 0.99$
- Current electoral system: $q = 0.67$

First scenario

No redistricting, 15% proportionality (678 deputies)



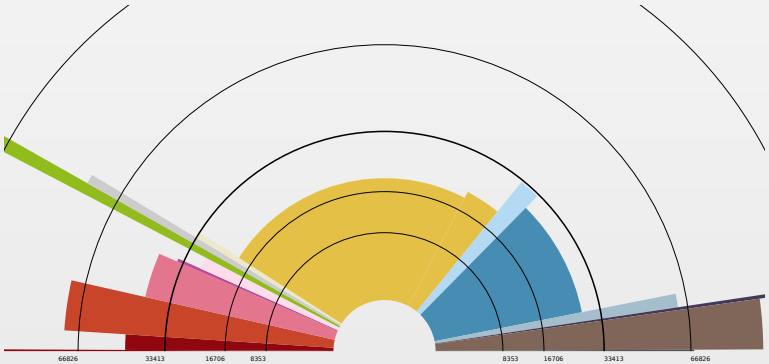
Purely majoritarian
($q = 0.67$)



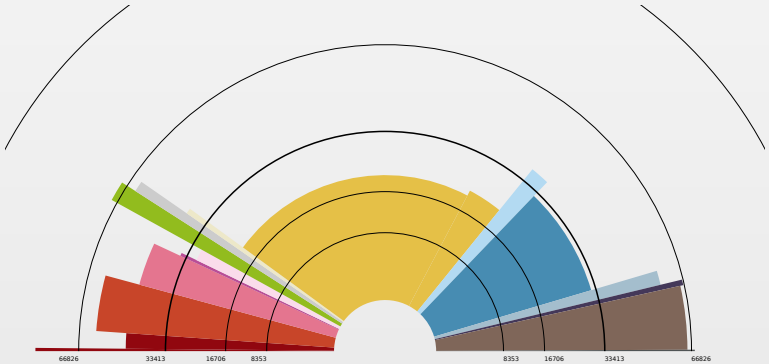
Additive mixed
($q = 0.72$)

First scenario

No redistricting, 15% proportionality (678 deputies)



Corrective mixed
($q = 0.75$)



Compensatory mixed
($q = 0.8$)

Second scenario

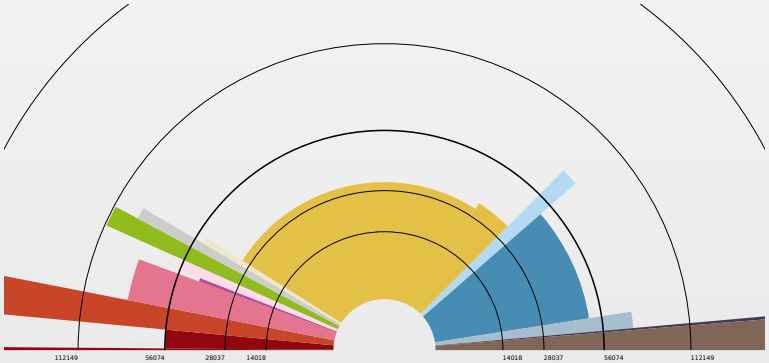
Purely majoritarian, reduction of the number of deputies:

- 577 deputies: $q = 0.67$
- 404 deputies: $q = 0.66$
- 364 deputies: $q = 0.65$
- 344 deputies: $q = 0.65$
- 323 deputies: $q = 0.65$
- 303 deputies: $q = 0.65$

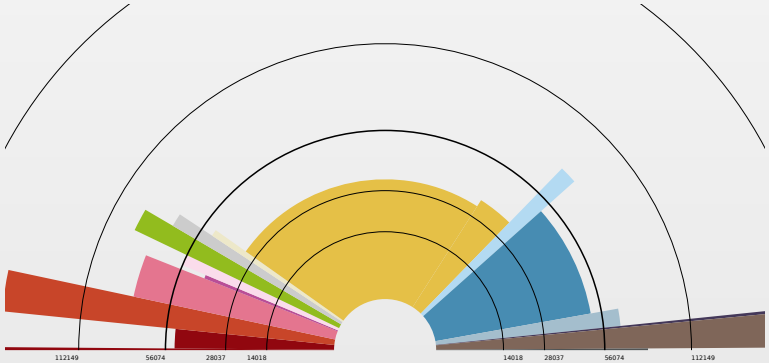
(With statistical generation of voters)

Third scenario

404 deputies, 15% proportionality (344 maj. + 60 prop.)



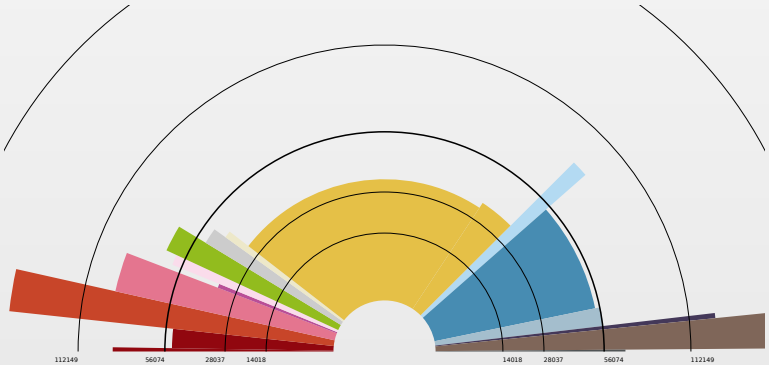
Mixed additive ($q = 0.73$)



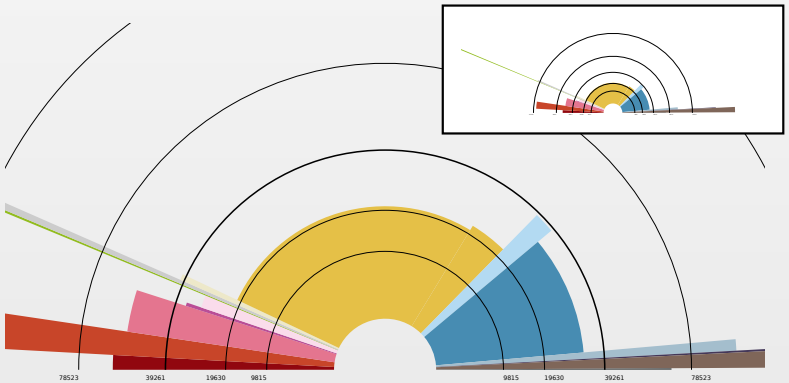
Mixed corrective ($q = 0.75$)

Third scenario

404 deputies, 15% proportionality (344 maj. + 60 prop.)



Mixed compensatory ($q = 0.77$)



Purely majoritarian, for comparison ($q = 0.67$)

Conclusion

A lot more scenarios tested... (see report for details)

http://recherche.noiraudes.net/resources/2018-05-28-rapport.pdf

Conclusion

A lot more scenarios tested... (see report for details)

http://recherche.noiraudes.net/resources/2018-05-28-rapport.pdf

Conclusions...

- Reducing the number of deputies strengthens the majority
- Introducing proportionality has the opposite effect

Not much more we can say for sure...