Conditional Importance Networks:
A Graphical Language for Representing Ordinal, Monotonic Preferences over Sets of Goods

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Preferences on combinatorial domains...

- Preferences on a set of alternatives $\Leftrightarrow$ defining a (reflexive) binary relation.
- Combinatorial set of alternatives $(\mathcal{D}_1 \times \cdots \times \mathcal{D}_n)$:
  - explicit representation: exponential size ;
  - $\Rightarrow$ compact representation language.
Introduction

Combinatorial domains

Configuration, voting, planning, resource allocation ...

A special case: Boolean combinatorial domains.

- Domains of values isomorphic to $\mathbb{B} = \{0, 1\}$.
- Well-suited for representing preferences on bundles of objects.
- Example: $\{o_1, o_2\} \succ \{o_3\}, \{o_3, o_4\} \succ \{o_3\}$.
- Preferences are very often (strictly) monotonic: $X \subsetneq Y \Rightarrow X \prec Y$. 
Outline of the talk

1. From (T)CP-nets to CI-nets
   - CP-nets
   - TCP-nets

2. Conditional Importance Networks
   - CI-nets: language and semantics
   - Expressivity
   - Computational issues
   - CI-nets and resource allocation
From (T)CP-nets to CI-nets

**CP-nets [Boutilier et al., 2004]**

**Example**

\[ x, y : z_1 > z_2 \rightarrow \text{“All other things being equal, if } X = x \text{ and } Y = y, \text{ then I prefer having } Z = z_1 \text{ than } Z = z_2”. \]

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CP-nets: A tool for representing and reasoning with conditional ceteris paribus preference statements.

From (T)CP-nets to CI-nets

CP-nets [Boutilier et al., 2004]

Example

\[ x, y : z_1 \succ z_2 \rightarrow \text{"All other things being equal, if } X = x \text{ and } Y = y, \text{ then I prefer having } Z = z_1 \text{ than } Z = z_2.\]

- CP-nets: \( a : b \succ \overline{b}; \)
- whereas we want: \( a : b \succ c. \)

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CP-nets: A tool for representing and reasoning with conditional ceteris paribus preference statements.
From (T)CP-nets to CI-nets

TCP-nets [Brafman et al., 2006]

- CP-nets enriched with (conditional) importance statements.
- **TCP-nets**: $a : b \triangleright c$

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**Brafman, R. I., Domshlak, C., and Shimony, S. E. (2006).**

On graphical modeling of preference and importance.

TCP-nets [Brafman et al., 2006]

- CP-nets enriched with (conditional) importance statements.
- TCP-nets: $a : b \triangleright c$
- ...but we also want: $a : bc \triangleright de$.

Conditional importance statement

Conditional importance statement: \( S^+, S^- : S_1 \triangleright S_2 \) (with \( S^+, S^- \), \( S_1 \) and \( S_2 \) pairwise-disjoint).

Example: \( \overline{ad} : b \triangleright ce \) implies for example \( ab \succ ace, abfg \succ acefg, \ldots \)

Cl-net

A Cl-net on \( \mathcal{V} \) is a set \( \mathcal{N} \) of conditional importance statements on \( \mathcal{V} \).
Semantics

A CI-net of 4 objects \( \{a, b, c, d\} \): \( a : d \succ bc, \overline{ad} : b \succ c, d : c \succ b \)
A CI-net of 4 objects \( \{a, b, c, d\} \): \( \{a : d \triangleright bc, a\overline{d} : b \triangleright c, d : c \triangleright b\} \)

Induced preference relation \( \succ_N \): the smallest preference monotonic relation compatible with all CI-statements.
Worsening flips

Worsening flip

\( \mathcal{V}_1 \sim \sim \mathcal{V}_2 \) is called a **worsening flip** wrt. \( \mathcal{N} \) if:
- either \( \mathcal{V}_1 \subseteq \mathcal{V}_2 \) (monotonicity flip);
- or they match a CI-statement in \( \mathcal{N} \) (CI-flip).

**Proposition (dominance)**

We have \( A \succ \succ \succ \mathcal{N} B \) if and only if there exists a sequence of worsening flips from \( A \) to \( B \) wrt. \( \mathcal{N} \).

**Proposition (satisfiability)**

A CI-net \( \mathcal{N} \) is satisfiable if and only if it does not possess any cycle of worsening flips.
Conditional Importance Networks

Expressivity

**Proposition**
CI-nets can express all strict monotonic preference relations on $2^\mathcal{V}$.

**Proof sketch:** for every $(X, Y)$ such that $X \succ Y$ and $X \nsubseteq Y$, add the CI-statement $(X \cap Y), \overline{(X \cup Y)} : X \setminus Y \succ Y \setminus X$.

**Proposition**
Full expressivity is lost as soon as:
(i) we do not allow positive preconditions;
(ii) we do not allow negative preconditions;
(iii) the cardinality of compared sets is bounded by a fixed integer.
Input: A satisfiable CI-net $\mathcal{N}$, a bundle $X$.
Question: Is $X$ non dominated for $\succ^{\mathcal{N}}$?

[Non-dominated]
Conditional Importance Networks

Optimization

[NON-DOMINATED]

**Input:** A satisfiable CI-net $\mathcal{N}$, a bundle $X$.

**Question:** Is $X$ non dominated for $\succ_\mathcal{N}$?

**Irrelevant:** the entire set is the only one non dominated set!
Conditional Importance Networks

Optimization

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**Irrelevant:** the entire set is the only one non dominated set!

More interesting:
- Constrained optimization
- Resource allocation
Input: A (satisfiable) CI-net $\mathcal{N}$, two bundles $X$ and $Y$.
Question: $X \succ_{\mathcal{N}} Y$ ?
Conditional Importance Networks

Dominance

[DOMINANCE]
Input: A (satisfiable) CI-net \( \mathcal{N} \), two bundles \( X \) and \( Y \).
Question: \( X \succ_{\mathcal{N}} Y \)?

Some bad news...

Proposition
[DOMINANCE] in satisfiable CI-nets is \textbf{PSPACE}-complete, even under any of these restrictions:

1. every CI-statement bears on singletons and has no negative preconditions;
2. every CI-statement bears on singletons and has no positive preconditions;
3. every CI-statement is precondition-free.
Conditional Importance Networks

**Dominance**

**Input:** A (satisfiable) CI-net \(\mathcal{N}\), two bundles \(X\) and \(Y\).

**Question:** \(X \succ_{\mathcal{N}} Y\) ?

Some good news...

**SCI-nets:** precondition-free, singleton-comparing CI-statements.

**Example:** \(\{a \triangleright c, b \triangleright c, e \triangleright d\}\).

**Proposition**

[DOMINANCE] in satisfiable SCI-nets is in \(\mathbb{P}\).
Satisfiability

**Input:** A CI-net $\mathcal{N}$.
**Question:** Is $\mathcal{N}$ satisfiable?
Conditional Importance Networks

Satisfiability

**[SATISFIABILITY]**

**Input:** A Cl-net $\mathcal{N}$.

**Question:** Is $\mathcal{N}$ satisfiable?

Some bad news...

**Proposition**

[[SATISFIABILITY]] for Cl-nets is **PSPACE**-complete.
Satisfiability

**Input:** A Cl-net $\mathcal{N}$.

**Question:** Is $\mathcal{N}$ satisfiable?

Some good news...

- **SATISFIABILITY** for SCI-nets is in $\mathbf{P}$.
- Two sufficient conditions for satisfiability: based on **acyclicity**.
Cl-nets and resource allocation

Cl-nets can be used to express fair division problems.

- **Objects**: \( V = \{a, b, c\} \).
- **Agents**:
  - \( N_1 = \{b : c \triangleright a, \bar{b} : a \triangleright c\} \);
  - \( N_2 = \{c \triangleright a, a \triangleright b\} \).
CI-nets and resource allocation

CI-nets can be used to express fair division problems.

- **Objects**: $\mathcal{V} = \{a, b, c\}$.
- **Agents**:
  - $\mathcal{N}_1 = \{b : c \succ a, \overline{b} : a \succ c\}$;
  - $\mathcal{N}_2 = \{c \succ a, a \succ b\}$

- $\langle 1 : a, 2 : bc \rangle$ is **not envy-free possible**.
- $\langle 1 : b, 2 : ac \rangle$ is **envy-free possible** but **not envy-free necessary**.
Conclusion

Summary and future work

- a new ordinal language based on conditional importance and on the Ceteris Paribus assumption;
- some satisfiability conditions;
- some investigations about expressivity of this language;
- some complexity results about this language.

Future work: We need to apply this to resource allocation and constrained optimization (some insights in the paper).

CI-nets: A Graphical Language for Representing Ordinal, Monotonic Preferences over Sets of Goods
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