# Picking Sequences for Resource Allocation

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# Fair division of indivisible goods...

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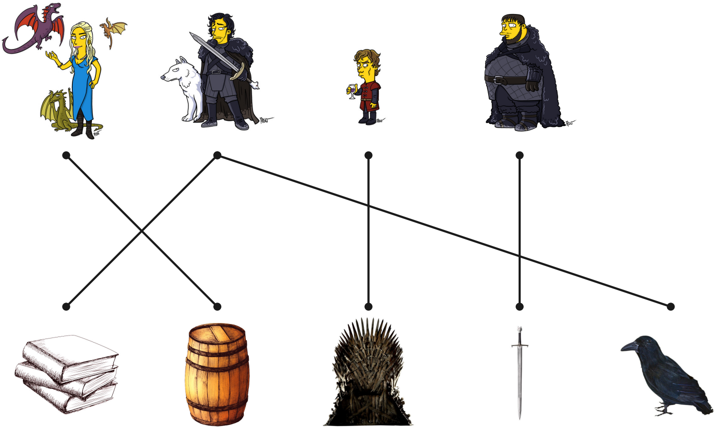

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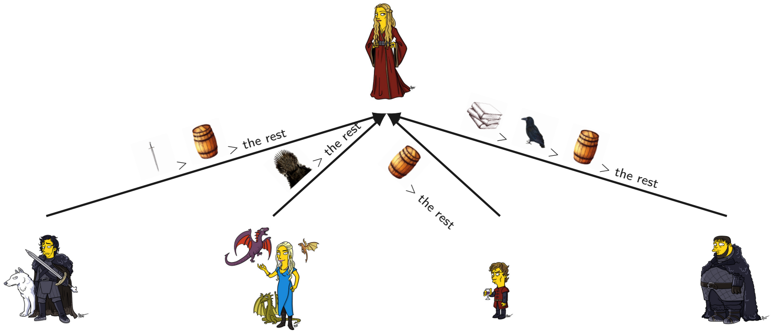

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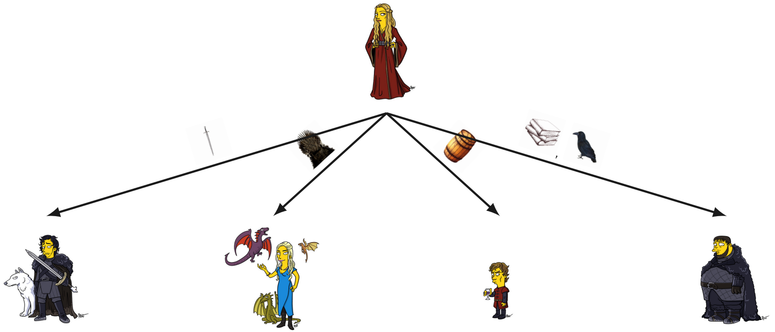

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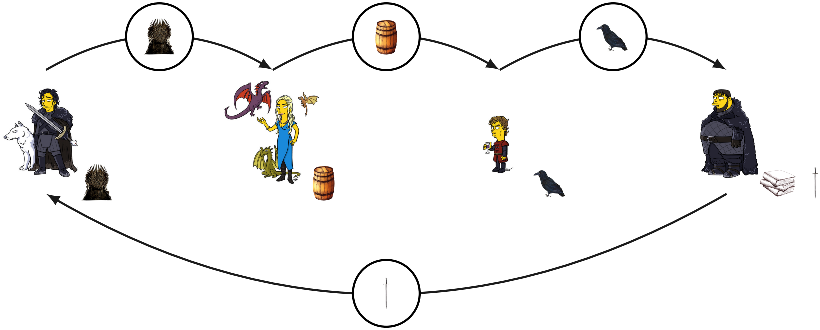

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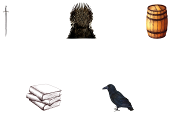

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In this presentation, we will focus on **picking sequences**



# More formally...

Ask the individuals to pick in turn their most preferred object among the remaining ones, according to some **predefined sequence**.

**Example**

3 individuals $A$, $B$, $C$, 6 items, sequence $ABCCBA \rightarrow A$ chooses first (and takes her preferred item), then $B$, then $C$, then $C$ again...

# Sequences

- Natural and simple protocol
- Used in practice
- Preference elicitation-free

- Board games
- Draft mechanisms (sport)
- Course allocation (Harvard Business School)
- ...

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Given a number of agents and a number of objects (+ some additional assumptions), what is the **fairest** sequence?

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Given a number of agents and a number of objects (+ some additional assumptions), what is the **fairest** sequence?

Today, we will mostly focus on the optimal sequence problem (+ touch upon some other notions like efficiency and strategyproofness)

# The optimal sequence problem

## What is the fairest sequence?

# Agents, objects, preferences. . .

- A set $\mathcal{O}$ of $p$ objects $\{1, \ldots, p\}$
- A set $\mathcal{N}$ of $n$ agents $\{A, B, \ldots, x\}$
- A central authority (CA) must find a **policy** (a sequence of agents) $\pi : \{1, \ldots, p\} \rightarrow \{A, B, \ldots, x\}$
- The central authority assumes that each agent $i$ has a (private) **ranking** $\succ_i$ over $\mathcal{O}$ (ex: $6 \succ 1 \succ 4 \succ 5 \succ 2 \succ 3$)

# Example

## Example

5 objects, 3 agents, $\pi = ABCCB\ldots$

- $A : 1 \succ 2 \succ 3 \succ 4 \succ 5$
- $B : 4 \succ 2 \succ 5 \succ 1 \succ 3$
- $C : 1 \succ 3 \succ 5 \succ 4 \succ 2$

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| $k$ | 0 |
|---|---|
| $s(A)_k^\pi$ | $\emptyset$ |
| $s(B)_k^\pi$ | $\emptyset$ |
| $s(C)_k^\pi$ | $\emptyset$ |
| $\mathcal{O}_k^\pi$ | $\emptyset$ |

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| $k$ | 0 | 1 | 2 |
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| $s(A)_k^\pi$ | $\emptyset$ | 1 | 1 |
| $s(B)_k^\pi$ | $\emptyset$ | $\emptyset$ | 4 |
| $s(C)_k^\pi$ | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $\mathcal{O}_k^\pi$ | $\emptyset$ | 1 | 14 |



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5 objects, 3 agents, $\pi = AB\mathbf{C}CB\ldots$

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| $s(A)_k^\pi$ | $\emptyset$ | 1 | 1 | 1 | 1 |
| $s(B)_k^\pi$ | $\emptyset$ | $\emptyset$ | 4 | 4 | 4 |
| $s(C)_k^\pi$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | 3 | 35 |
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- $B : 4 \succ 2 \succ 5 \succ 1 \succ 3$
- $C : 1 \succ 3 \succ 5 \succ 4 \succ 2$

| $k$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $s(A)^\pi_k$ | $\emptyset$ | 1 | 1 | 1 | 1 | 1 |
| $s(C)^\pi_k$ | $\emptyset$ | $\emptyset$ | 4 | 4 | 4 | 42 |
| $s(C)^\pi_k$ | $\emptyset$ | $\emptyset$ | $\emptyset$ | 3 | 35 | 35 |
| $\mathcal{O}^\pi_k$ | $\emptyset$ | 1 | 14 | 143 | 1435 | 14352 |

## Scoring functions

We only have rankings over objects. . .

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Two natural assumptions:

1. **Scoring:** We have a common **scoring function** $g : \{1, \ldots, p\} \mapsto \mathbb{N}$ mapping each rank to a utility.
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# Back to the example

## Example

5 objects, 3 agents, $\pi = ABCCB\ldots$

- $A : \mathbf{1} \succ 2 \succ 3 \succ 4 \succ 5$
- $B : \mathbf{4} \succ \mathbf{2} \succ 5 \succ 1 \succ 3$
- $C : 1 \succ \mathbf{3} \succ \mathbf{5} \succ 4 \succ 2$

With $\pi$, agent $A$ gets 1, agent $B$ gets 24, agent $C$ gets 35

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With $\pi$, agent $A$ gets 1, agent $B$ gets 24, agent $C$ gets 35

- **Borda:** $u_A(\pi) = 5$; $u_B(\pi) = 5 + 4 = 9$; $u_C(\pi) = 4 + 3 = 7$.
- **lexicographic:** $u_A(\pi) = 16$; $u_B(\pi) = 24$; $u_C(\pi) = 12$.
- **QI:** $u_A(\pi) = 1 + 4\varepsilon$; $u_B(\pi) = 2 + 7\varepsilon$; $u_C(\pi) = 2 + 5\varepsilon$.



# Social welfare

We use a **collective utility function** to aggregate the individual utilities.

Two well-known functions:

- **utilitarian:** $F(u_A, \dots, u_x) = \sum_{i \in \mathcal{N}} u_i$;
- **egalitarian:** $F(u_A, \dots, u_x) = \min_{i \in \mathcal{N}} u_i$;
- (We will also speak about **Nash** in the second part: $F(u_A, \dots, u_x) = \prod_{i \in \mathcal{N}} u_i$).

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The procedure is elicitation-free. . .

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The CA has a **prior probability** on the preference profile:

- **Full independence** (FI): each profile $R = \langle \succ_A, \dots, \succ_x \rangle$ is equally probable
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Expected individual and collective utilities:

$$EU^{\Psi}(i, \pi) = \mathbb{E}_{R \sim \Psi}[u^R(i, \pi)] = \sum_{R \in Prof(\mathcal{N}, \mathcal{O})} Pr_{\Psi}(R) \times u_i^R(\pi, R).$$

$$ESW_F^{\Psi}(\pi) = F(EU^{\Psi}(1, \pi), \dots, EU^{\Psi}(n, \pi)).$$

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**Example**

5 objects, 3 agents, $\pi = ABCCB$, $g = g_{Borda}$, full independence.
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*What is agent $C$'s expected utility with this sequence ?*

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$$EU^{FI}(3, \pi) = \frac{1}{\binom{5}{2}} \times (3 + 2)$$

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*What is agent $C$'s expected utility with this sequence ?*
$C$'s preferences: $? \succ ? \succ ? \succ ? \succ ?$

$$EU^{FI}(3, \pi) = 0.5 + \frac{1}{\binom{5}{2}} \times (4 + 2)$$

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*What is agent C's expected utility with this sequence ?*

$C$'s preferences: $? \succ ? \succ ? \succ ? \succ ?$

$$EU^{FI}(3, \pi) = 0.5 + 0.6 + \frac{1}{\binom{5}{2}} \times (5 + 2)$$

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*What is agent $C$'s expected utility with this sequence ?*

$C$'s preferences: $? \succ ? \succ ? \succ ? \succ ?$

$$EU^{FI}(3, \pi) = 0.5 + 0.6 + 0.6 + \frac{2}{\binom{5}{2}} \times (4 + 3)$$

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$C$'s preferences: $? \succ ? \succ ? \succ ? \succ ?$

$$EU^{FI}(3, \pi) = 0.5 + 0.6 + 0.6 + 1.4 + 1.6 + 2.7 = \mathbf{7.5}$$

# Summary

- **Instance:**
  - a number of agents $n$
  - a number of objects $p$
  - a scoring function $g$
  - a prior (*i.e* a correlation assumption) $\Psi \in \{FC, FI\}$
  - a collective utility function $F$
- **Question:**
  - *What is the policy* $\pi$ *maximizing* $ESW_F^{\Psi}(\pi)$*, under correlation profile* $\Psi$*?*

# Some general results

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Sequential allocation is **NP**-complete.

(Reduction from [PARTITION])

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## 1. Full correlation

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All policies have the same expected value!

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Sequential allocation is **NP**-complete.

(Reduction from [PARTITION])

What about...

- ... lexicographic scoring ?
- ... quasi-indifference scoring ?
- ... Borda scoring ?

# Lexicographic scoring

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\succ_i$ | 6 | $\succ$ | 1 | $\succ$ | 4 | $\succ$ | 5 | $\succ$ | 2 | $\succ$ | 3 |
| lexicographic | 32 | $\gg$ | 16 | $\gg$ | 8 | $\gg$ | 4 | $\gg$ | 2 | $\gg$ | 1 |

## Egalitarian CUF (min)

**Optimal policies:** $\sigma(A)\sigma(B)\ldots\sigma(x)\sigma(x)^{p-n}$ (where $\sigma$ is a permutation of $\{A, B, \ldots, x\}$)

# Lexicographic scoring

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**Example:** $\pi = ABCCCC$

- $EU^{FC}(1, \pi) = 32$
- $EU^{FC}(2, \pi) = 16$
- $EU^{FC}(3, \pi) = 8 + 4 + 2 + 1 = 15$

# Borda scoring

| | | | | | | |
|---|---|---|---|---|---|---|
| $\succ_i$ | 6 | 1 | 4 | 5 | 2 | 3 |
| Borda | 6 | 5 | 4 | 3 | 2 | 1 |

# Borda scoring

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**Egalitarian CUF (min)**

This is **polynomial** in $p$.

**Hint:** The number of possible utility values is bounded for an agent $(p(p+1)/2) \rightarrow$ use a dynamic programming algorithm.

# QI scoring

| $\succ_i$ | 6 | 1 | 4 | 5 | 2 | 3 |
|---|---|---|---|---|---|---|
| Quasi-Indifference | $1+5\varepsilon$ | $1+4\varepsilon$ | $1+3\varepsilon$ | $1+2\varepsilon$ | $1+\varepsilon$ | $1$ |

**Egalitarian CUF (min)**

Comes down to solving the Borda case!

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Comes down to solving the Borda case!

**Intuition:**

- let $m = \lfloor \frac{p}{n} \rfloor$ and $q = p - nm$
- Optimal policies: $\pi = \underbrace{AABB}_{n-q \text{ agents}}\overbrace{CCCDDD}^{q \text{ agents}}$ and $\pi' = \underbrace{ABBA}_{n-q \text{ agents}}\overbrace{CCCDDD}^{q \text{ agents}}$
- The $q$ last agents are OK $\rightarrow u \geq m+1$
- The $n-q$ first agents: $u = m + x \cdot \varepsilon$ ($x \rightarrow$ Borda)

# A complex problem. . .

## 2. Full independence

# A complex problem. . .

**2. Full independence**

**Conjecture (2011)**

Computing the expected utility of a sequence is **NP**-complete.
Computing the **optimal** sequence probably harder.

# Results

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# Results

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Computing the **optimal** sequence probably harder.

- the **alternating policy** (*ABABABAB*...) is optimal for Borda, utilitarian social welfare
- complexity unknown for other social welfare and scoring functions (**NP**-hardness conjectured)

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# Some examples

**Assumptions:** Full independence, egalitarian CUF, Borda scoring function.

| $p$ | $n = 2$ | $n = 3$ |
|---|---|---|
| 4 | | |
| 5 | | |
| 6 | | |
| 8 | | |
| 10 | | |

# Some examples

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| $p$ | $n = 2$ | $n = 3$ |
|---|---|---|
| 4 | ABBA | ABCC |
| 5 | AABBB | ABCCB |
| 6 | ABABBA | ABCCBA |
| 8 | ABBABAAB | AACCBBCB |
| 10 | ABBAABABBA | ABCABBCACC |

# A small digression about strategical issues (manipulation)

**Is the protocol strategy-proof?**

# Manipulation?

- A set $\mathcal{O}$ of $p$ objects $\{1, \ldots, p\}$
- A set $\mathcal{N}$ of $n$ agents $\{A, B, \ldots, x\}$
- A central authority (CA) has chosen a **policy** $\pi$ and will execute it
- The agents have their own private preferences $\rightarrow$ **picking strategy**.

# Manipulation?

## Example

2 agents, 4 objects:

- $A$: $1 \succ 2 \succ 3 \succ 4$
- $B$: $2 \succ 3 \succ 4 \succ 1$

Sequence $\pi = ABBA \rightarrow \{14|23\}$.

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*What if A knows B's preferences and acts maliciously?*

She can manipulate by picking 2 instead of 1 at first step $\rightarrow \{12|34\}$.

## More formally

- A set $\mathcal{O}$ of $p$ objects $\{1, \ldots, p\}$
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- A **policy** $\pi$
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**The cheating agent ($A$) knows:**

- the sequence
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**She wants:**

- to get the best bundle she can get.

**Her only possible cheating actions:**

- choose at given steps **not** to pick her preferred objects.

# Bad news...

**(Folk?) theorem**

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Two possible actions to prevent manipulation as a mechanism designer:

- Impose strategyproofness
- Use complexity as a barrier to manipulation

## Strategyproof picking sequences

**What can we do if we impose strategyproofness in picking sequences?**

# Non interleaving sequences

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$\rightarrow$ We can compensate late arrival by higher number of goods picked.

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- For regular (general) sequences, finding the fairest sequence was complex in most cases
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**Main result**

For $\Psi = FI - FC_\lambda$, for any $\lambda \in [0,1]$, any scoring function, and any $F \in \{\min, +, \times\}$, we can find in **polynomial time** the vector $(k_1, \ldots, k_n)$ that maximizes $ESW_F^\Psi(k)$.

Note: $FI - FC_\lambda$ is a mixture. The pref. profile is sampled according to $FI$ with probability $\lambda$ and FC with probability $1 - \lambda$.

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You may have already heard of non-interleaving picking sequences, that have some similarities with **sequential/serial dictatorship**

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You may have already heard of non-interleaving picking sequences, that have some similarities with **sequential/serial dictatorship**

- Serial dictatorship:
  - a fixed sequence of agents
  - the 1st one takes her best bundle
  - then the 2nd one takes her best bundle among the remaining items
  - and so on...
- Sequential dictatorship: same, but the sequence depends on the preference profile [Pápai, 2001]
- Sequential quota choice function: same as non-interleaving picking sequences [Pápai, 2000]

**Pápai, S. (2000).**
original papers : Strategyproof multiple assignment using quotas.
*Review of Economic Design*, 5(1):91–105.

**Pápai, S. (2001).**
Strategyproof and Nonbossy Multiple Assignments.
*Journal of Public Economic Theory*, 3(3):257–271.

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Note: Here, $v_1$ only depends on $k_1$ and is easy to compute (the utility of the $k_1$ best objects)

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Note: Here, $v_2$ depends on $k_2$ but not only...

# Computation of $v$

## Key property

If $\Psi = FI - FC_\lambda$, for any $\lambda$, for any agent $i$, $v_i$ only depends on:

- # of items received by $i \rightarrow k_i$
- # of items picked before $\rightarrow t$

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- If $\Psi = FC$: $v(k,t) = \sum_{i=t+1}^{t+k} g(i)$
- If $\Psi = FI$, $v(k,t)$ can be computed by dynamic programming by carefully analyzing the probability that a given item has been picked before.

# Some examples

Example of values of $v(k, t)$ for $\Psi = FI$:

| $k \backslash t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 7 | 6.86 | 6.67 | 6.4 | 6 | 5.33 | 4 | - |
| 2 | 13 | 12.57 | 12 | 11.2 | 10 | 8 | - | - |
| 3 | 18 | 17.14 | 16 | 14.4 | 12 | - | - | - |
| 4 | 22 | 20.57 | 18.67 | 16 | - | - | - | - |
| 5 | 25 | 22.86 | 20 | - | - | - | - | - |
| 6 | 27 | 24 | - | - | - | - | - | - |
| 7 | 28 | - | - | - | - | - | - | - |

Note: the approach is similar to [Kalinowski et al., 2013]

**Kalinowski, T., Narodytska, N., and Walsh, T. (2013).**
A social welfare optimal sequential allocation procedure.
In *Proceedings of IJCAI 2013*.

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$$\widehat{ESW}_F^{\Psi}(i,\ell) = \max_{k\in\{0,\ldots,m-\ell\}} F\left(v(k,l), \widehat{ESW}_F^{\Psi}(i+1,l+k)\right)$$

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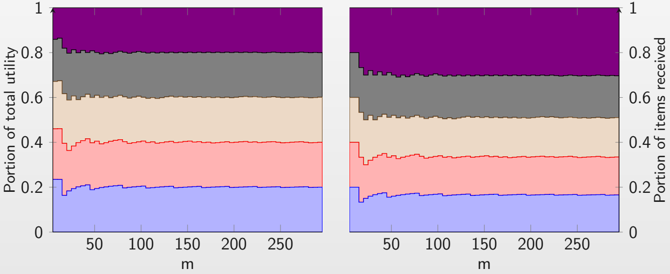
$$\widehat{ESW}_F^{\Psi}(n, \ell) = v(m - \ell, \ell)$$

Computable in polynomial time using dynamic programming ($O(nm^2)$ once the values $v(k, t)$ have been computed).

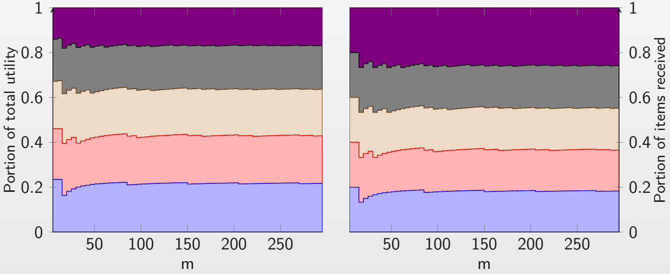
# Some results (egalitarianism)

5 agents, varying # of goods, Borda scoring vector, $\Psi = FI$, $F = \min$



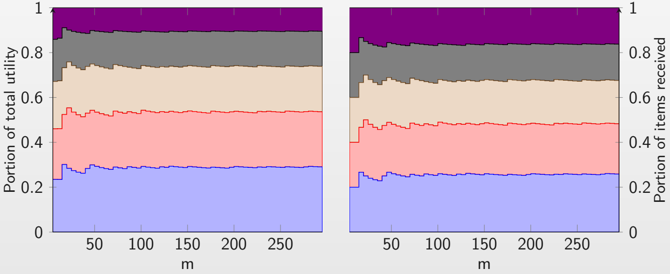
# Some results (Nash)

5 agents, varying # of goods, Borda scoring vector, $\Psi = FI$, $F = \times$



# Some results (utilitarianism)

5 agents, varying # of goods, Borda scoring vector, $\Psi = FI$, $F = +$




Do you see anything strange?

# Computational barriers to manipulation

**Can we rely on computational complexity to prevent manipulation?**

# Complexity as a barrier to manipulation

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**Getting a subset for sure**

We can answer to that constructively in polynomial time!

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## Getting a subset for sure

We can answer to that constructively in polynomial time!

### Idea:

- two agents: pick the objects in $\mathcal{S}$ in the same order of $\succ_B$
- more agents:
  - transform agents 2 to $m-1$ into a single (fake) agent
  - apply the algorithm for 2 agents



# General manipulation problem

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**Idea:** Greedily build the optimal achievable subset:

- Find the best object $i$ such that $\{i\}$ is achievable;
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If the manipulator has additive preferences, the optimal manipulation can be computed in polynomial time.

Only works **for two agents**!

# General manipulation problem

**Result [Aziz et al., 2017]**

If the manipulator has additive preferences, the optimal manipulation problem is **NP**-complete.

(reduction from [3-SAT])

- Is there a manipulation that yields a better utility than the truthful report? $\rightsquigarrow$ **NP**-complete
- Not true anymore for binary utilities and (ordinal) responsive set extension.

**Aziz, H., Bouveret, S., Lang, J., and Mackenzie, S. (2017).**
Complexity of manipulating sequential allocation.
In *Proceedings of the 31st AAAI conference on Artificial Intelligence (AAAI'17).*

# Coalitional Manipulation

**Example**

3 agents, 6 objects:

- $A$: $1 \succ 2 \succ 5 \succ 4 \succ 3 \succ 6$
- $B$: $1 \succ 3 \succ 5 \succ 2 \succ 4 \succ 6$
- $C$: $2 \succ 3 \succ 4 \succ 1 \succ 5 \succ 6$

Sequence $\pi = ABCABC \rightarrow \{15|34|26\}$.

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Sequence $\pi = ABCABC \rightarrow \{15|34|26\}$.

- If $A$ and $B$ manipulate alone, they cannot do better
- If they cooperate, they can get $\{12|35|46\}$, which is strictly better.

# Coalitional Manipulation: Results

Three kinds of manipulation considered here:

- No post-allocation trade allowed between the manipulators
- Post-allocation exchange of goods allowed between the manipulators
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## Results:

- No post-allocation trade allowed between the manipulators $\to$ **NP**-complete [PARTITION]
- Post-allocation exchange of goods allowed between the manipulators $\to$ **NP**-complete [PARTITION]
- Post-allocation exchange of goods + side-payments allowed $\to$ polynomial (comes down to manipulation by a single agent)

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**Game Theory** (Subgame Perfect Nash Equilibrium)



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**Game Theory** (Subgame Perfect Nash Equilibrium)

- Two agents and additive utilities, precise characterization of the result of every SPNE $((rev(\succ_2), rev(\succ_1), rev(\pi)))$ [Kalinowski et al., 2013, Kohler and Chandrasekaran, 1971].
- Unbounded number of agents: **PSPACE**-hard [Kalinowski et al., 2013].

**Kalinowski, T., Narodytska, N., Walsh, T., and Xia, L. (2013).**
Strategic behavior when allocating indivisible goods sequentially.
In *Proceedings of AAAI'13.*

**Kohler, D. A. and Chandrasekaran, R. (1971).**
A class of sequential games.
*Operations Research*, 19(2):270–277.

# Conclusion

## A take-away message?

# Conclusion

- A simple and intuitive sequential allocation procedure (actually already known in sparse litterature)
- Finding the best policy?
  - Full Correlation case well understood
  - Full Independence: partial (hardness) results
- Strategical issues:
  - Non-interleaved sequences: strategyproof, and can be fair!
  - Complexity as a barrier to (individual and coalitional) manipulation

# A note about efficiency

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As soon as we constrain serial dictatorship, we lose Pareto-efficiency.
But:

- Any Pareto-efficient allocation is sequenceable
- The converse is true for
  - **quantity-monotonic** preferences [Pápai, 2000]
  - **lexicographic** preferences [Hosseini and Larson, 2019]
- Sequenceable allocations correspond to a weak form of Pareto-optimal ones
- $\rightarrow$ Sequenceability $\approx$ weak (local) form of efficiency

**Hosseini, H. and Larson, K. (2019).**
Multiple assignment problems under lexicographic preferences.
In *Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS '19*, pages 837–845.

**Pápai, S. (2000).**
original papers : Strategyproof multiple assignment using quotas.
*Review of Economic Design*, 5(1):91–105.

## Very nice properties...

A simple protocol, but with nice features:

- "locally" efficient
- efficient with respect to cycle deals
- guarantees envy-freeness up to one good
- gives good approximation of social welfare
- also gives good approximation of other fairness properties (*e.g.* max-min share)