Conditional Importance Networks:

A Graphical Language for Representing Ordinal, Monotonic Preferences over Sets of Goods

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International Joint Conference on Artificial Intelligence, July 16, 2009

Preferences on combinatorial domains...

- Preferences on a set of alternatives ⇔ defining a (reflexive) **binary relation**.
- Combinatorial set of alternatives ($\mathcal{D}_1 \times \cdots \times \mathcal{D}_n$):
  - explicit representation: exponential size ;
  - ⇒ **compact representation language**.

Combinatorial domains

Configuration, voting, planning, **resource allocation** ...

A special case: **Boolean combinatorial domains**.

- Domains of values isomorphic to $\mathbb{B} = \{0, 1\}$.
- Well-suited for representing preferences on **bundles of objects**.
- **Example**: $\{o_1, o_2\} \succ \{o_3\}$, $\{o_3, o_4\} \succ \{o_3\}$.
- Preferences are very often (strictly) **monotonic**: $X \subsetneq Y \Rightarrow X \prec Y$.

Outline of the talk

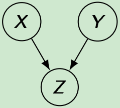
1. **From (T)CP-nets to CI-nets**
   - CP-nets
   - TCP-nets

2. **Conditional Importance Networks**
   - CI-nets: language and semantics
   - Expressivity
   - Computational issues
   - CI-nets and resource allocation

CP-nets [Boutilier et al., 2004]

Example

$x, y : z_1 \succ z_2 \longrightarrow$ "All other things being equal, if $X = x$ and $Y = y$, then I prefer having $Z = z_1$ than $Z = z_2$".



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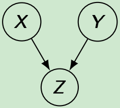

- **CP-nets**: $a : b \rhd \overline{b}$;
- whereas we want: $a : b \rhd c$.

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TCP-nets [Brafman et al., 2006]

- CP-nets enriched with (conditional) importance statements.
- **TCP-nets**: $a : b \triangleright c$

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TCP-nets [Brafman et al., 2006]

- CP-nets enriched with (conditional) importance statements.
- **TCP-nets**: $a : b \rhd c$
- ... but we also want: $a : bc \rhd de$.

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CI-nets: the language

Conditional importance statement

Conditional importance statement: $\mathcal{S}^+, \mathcal{S}^- : \mathcal{S}_1 \triangleright \mathcal{S}_2$ (with $\mathcal{S}^+$, $\mathcal{S}^-$, $\mathcal{S}_1$ and $\mathcal{S}_2$ pairwise-disjoint).

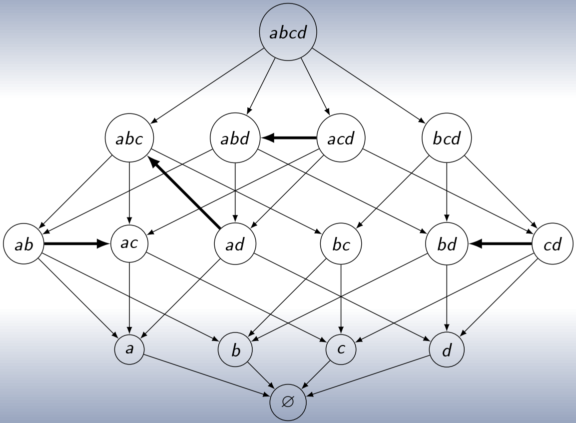
Example: $a\overline{d} : b \triangleright ce$ implies for example $ab \succ ace$, $abfg \succ acefg$, $\dots$

CI-net

A CI-net on $\mathcal{V}$ is a set $\mathcal{N}$ of conditional importance statements on $\mathcal{V}$.

Semantics

A CI-net of 4 objects $\{a, b, c, d\}$: $\{a : d \triangleright bc, a\overline{d} : b \triangleright c, d : c \triangleright b\}$



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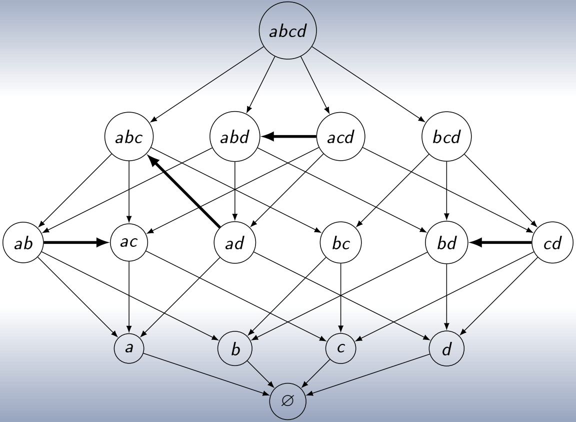

Induced preference relation $\succ_{\mathcal{N}}$: the smallest preference **monotonic** relation compatible with **all CI-statements**.

Worsening flips

Worsening flip

$\mathcal{V}_1 \rightsquigarrow \mathcal{V}_2$ is called a **worsening flip** wrt. $\mathcal{N}$ if:

- either $\mathcal{V}_1 \subseteq \mathcal{V}_2$ (monotonicity flip);
- or they match a CI-statement in $\mathcal{N}$ (CI-flip).

Proposition (dominance)

We have $A \succ_{\mathcal{N}} B$ if and only if there exists a sequence of worsening flips from A to B wrt. $\mathcal{N}$.

Proposition (satisfiability)

A CI-net $\mathcal{N}$ is satisfiable if and only if it does not possess any cycle of worsening flips.

Expressivity

Proposition

CI-nets can express all strict monotonic preference relations on $2^{\mathcal{V}}$.

Proof sketch: for every (X, Y) such that $X \succ Y$ and $X \not\supset Y$, add the CI-statement $(X \cap Y), (\overline{X \cup Y}) : X \setminus Y \triangleright Y \setminus X$.

Proposition

Full expressivity is lost as soon as:

(i) we do not allow positive preconditions;

(ii) we do not allow negative preconditions;

(iii) the cardinality of compared sets is bounded by a fixed integer.

Optimization

[NON-DOMINATED]

Input: A satisfiable CI-net $\mathcal{N}$, a bundle X.
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More interesting:

- **Constrained optimization**
- **Resource allocation**

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Some bad news...

Proposition

[DOMINANCE] in satisfiable CI-nets is **PSPACE**-complete, even under any of these restrictions:

1. every CI-statement bears on singletons and has no negative preconditions;
2. every CI-statement bears on singletons and has no positive preconditions;
3. every CI-statement is precondition-free.

Dominance

[DOMINANCE]

Input: A (satisfiable) CI-net $\mathcal{N}$, two bundles X and Y.
Question: $X \succ_{\mathcal{N}} Y$?

Some good news...
SCI-nets: precondition-free, singleton-comparing CI-statements.
Example: $\{a \rhd c, b \rhd c, e \rhd d\}$.

Proposition

[DOMINANCE] in satisfiable SCI-nets is in P.

Satisfiability

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Some good news...

- [SATISFIABILITY] for SCI-nets is in P.
- Two **sufficient** conditions for satisfiability: based on **acyclicity**.

CI-nets and resource allocation

CI-nets can be used to express fair division problems.

- **Objects**: $\mathcal{V} = \{a, b, c\}$.
- **Agents**:
  - $\mathcal{N}_1 = \{b : c \triangleright a, \overline{b} : a \triangleright c\}$;
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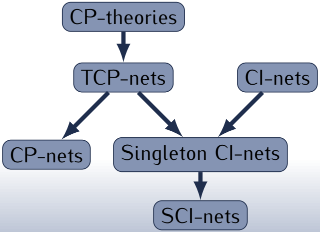
- $\langle 1 : a, 2 : bc \rangle$ is **not envy-free possible**.
- $\langle 1 : b, 2 : ac \rangle$ is **envy-free possible** but **not envy-free necessary**.

Summary and future work

- a new ordinal language based on conditional importance and on the Ceteris Paribus assumption;
- some satisfiability conditions;
- some investigations about expressivity of this language;
- some complexity results about this language.



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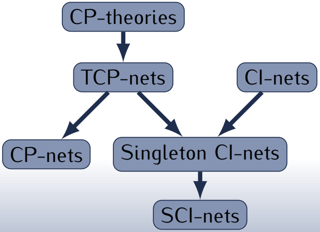

- Future work: We need to apply this to resource allocation and constrained optimization (some insights in the paper).